Inflation Targeting, Exchange-Rate Targeting Monetary Policies in an Open New Keynesian DSGE Model

Hsu Chen-Min** · Tsai Yue-Kun***

Recently the Fed’s monetary policy target interest rate is paid attention. How monetary policy should be publicly disclosed is related to the monetary policy strategy, including inflation targeting, exchange-rate targeting and the Taylor rule. A dynamic stochastic general equilibrium (DSGE) New Keynesian open macroeconomic model is setup with a Bayesian approach to estimate the model parameters using the Taiwan data. The effects of central bank’s different targeting policies on the economy with different shocks are investigated. We found that the economic welfare of the policy target adopted by the central bank depends on the real or nominal shocks.

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1. INTRODUCTION

In recent years both academic and practice circles have greatly attached importance to study the issue of central bank’s monetary policy targeting. In particular, after the US economy recovered from the 2007-2016 Great Recession, people have been increasingly paying more attention to the Fed’s monetary policy target interest rate and when the FOMC will raise the targeting federal funds rate. How monetary policy should be publicly disclosed in order to make overall economy achieve a higher social welfare is related to the issue of the monetary policy strategy, including inflation targeting, exchange-rate targeting and the Taylor rule (see Mishkin and Eakins (2015, Ch. 10), Miles, Scott, and Breeden (2012, Ch. 13)). As pointed out by Geraats (2002), in 1990 New Zealand, Canada, the United Kingdom and Switzerland implemented explicit inflation targeting to peg inflation at a fixed level, and their central bank had allowed monetary policy, econometric models and prediction results known to public, and thus showed the transparency of monetary policy of these countries.

Contradicting to the transparent monetary policy adopted by countries such as the United Kingdom, central banks of other countries did not simply take price inflation as the execution basis of monetary policy, but would refer to aggregate output or other economic situations when conducting monetary policy, for example the US Federal Reserve. Do the central banks make policy adjustments only according to price stability objective? Or do they need to bear the responsibility of stimulating economy? The central banks tend to make conflict decision when it considered too many objectives.

Conflict decision of monetary policy is due to the policy trade-off faced by the central bank when it weighs multiple economic objectives. In fact, if central bank is more discretionary in monetary policy objectives, then its interest rate policy will be less explicit and ambiguous than that of pure inflation objective.

In addition, as pointed out by Cukierman (2002), central bank’s asymmetric behavior of responding to economic situation is an even less
transparent policy behavior. Central bank’s asymmetric behavior can be described as the following: the central bank’s degrees of sensitivity are not the same on recession and boom. For example: the government has the responsibility of preventing excessive economic downturn and tries to find ways to stimulate domestic output, when the economy is in recession. The government may urge the central bank to respond to economic recession through a variety of channels. At this time central bank is very sensitive to output gap. But when economic situation is overly booming, the central bank will less consider to adjust aggregate output back to natural status due to smaller political pressure from the administration. At this time the central bank’s degree of feelings to output gap is less sensitive than when economic situation is in recession. Thus, the central bank’s policy response to deviations from either side of full employment is quite different.

Cukierman (2002) showed that central bank’s asymmetric behavior often results in greater economic losses.

Observing multiple objectives leading to trade-off faced by central bank’s decision-making, an important issue comes out, i.e. whether central bank’s decision-making would result in social welfare loss. The main goal of this paper aims to explore: whether different kinds of the central bank’s policy targeting may affect the economy and thus result in extra social welfare losses. We found that the economic welfare of the policy target adopted by the central bank depends on the types of shocks. That is, whether the shock is real or nominal is crucial to the welfare of monetary policy implementation.

In this paper, a dynamic stochastic general equilibrium (DSGE) model is utilized to construct a small New Keynesian open macroeconomic model. Following Gali and Gertler (1999), Gali and Monacelli (2005), Walsh (2010, Ch. 8), this model contains a certain degree of price rigidity, and two types of firms with forward looking and backward looking expectations. Then we used a Bayesian approach to estimate the parameters of the model using Taiwan data. We put several shocks onto a number of sectors of the model, and examined the effects of central bank’s different targeting policies on the
We found that the economic welfare of the policy target adopted by the central bank depends on the types of shocks. That is, whether the shock is real or nominal is crucial to the welfare of monetary policy implementation.

Our study differs from the literature of the analysis of flexible inflation targeting in a DSGE framework in that both the theoretical analysis and empirical estimation, as well as the model simulation and calibration of monetary policy analysis, are consistently unified together. This consistency and unification have made our study distinct from the current related works, e.g. Clarida, Gali, and Gertler (1999), Woodford (2003), Gali and Gertler (2007), Gali (2008), Walsh (2010, Ch. 8), Walsh (2009a), Walsh (2009b). In particular, e.g. Walsh (2009b), these works usually take a rigorous analysis for their theoretical model. However, when the policy analyses of monetary transmission process are illustrated, they utilized a VAR model to show impulse responses of shocks.

The rest of the paper is organized as follows. Section 2 reviews the related literature. Section 3 presents the model to describe the central bank's behavior and the economic structure of a small open economy with New Keynesian DSGE feature. The shocks are depicted and the coefficients of the model are estimated. Section 4 compares the economic welfare of various monetary policy targeting rules. Section 5 concludes.

## 2. LITERATURE REVIEW

### 2.1. Monetary Policy Rule

#### 2.1.1. Inflation forecast targeting and inflation targeting rule

The research of monetary policy rule (or monetary policy reaction function) had been very popular in 1990s and 2000s, since Taylor (1993) proposed the so-called Taylor (interest rate) rule which described the actual behavior of the federal fund rate rather well. However as pointed by
Svensson (1997), the Taylor (1993) rule for the federal funds rate was an instrument rule. The Taylor (1993) rule and the McCallum rule for the monetary base (see McCallum (1990)) were two prominent instrument rules and they were not endogenous optimal reaction functions expressing the instrument in terms of current information. An instrument rule, for example the traditional Taylor (1993) rule, directly specified the reaction function for the instrument (i.e., the federal fund rate) in terms of current information. Svensson (1997) had therefore set up a quadratic-linear optimization model to describe the central bank behavior of interest rate policy rule, which was an endogenous optimal linear reaction function. The objective function was an expected multi-period quadratic loss function and the structure of the economy was represented by a VAR model of inflation, output rate and exogenous variable. Using the two-year inflation forecast targeting as an explicit intermediate target, Svensson could derive the central bank’s optimal reaction function which was of the same form as the Taylor (1993) rule, except that it also depended on the exogenous variable. He further pointed that the central bank’s inflation forecast appeared to be an ideal intermediate target.

Since inflation forecast targeting uses all relevant information for predicting future inflation, this information may include money stock growth, exchange rate depreciation rate, and other macro variables. If the inflation forecast equals to the inflation target, it is inflation targeting. And when the inflation forecast is set to be exchange rate depreciation rate, it becomes exchange rate targeting. Furthermore, it will be equivalent to money growth targeting, if the inflation forecast equals to money growth (see Svensson (1997)).

**2.1.2. Asymmetric monetary policy rule**

Svensson (1997) monetary policy rule is a linear monetary-policy reaction function, although it is an endogenous function and a target rule. It is derived from a linear-quadratic framework, with a quadratic loss function and a linear dynamic system describing the economy. Later on there were a
series of research challenging the assumptions underlying the linear-quadratic framework. First, Cukierman (2002) specified the loss function as dependent on the state of the business cycle. Surico (2007) adopted an asymmetric linear specification of the loss function. Secondly, Dolado, Maria-Dolores, and Naveira (2005) relaxed the linear assumption of the Phillips curve or the aggregate supply relation. Thirdly, as pointed by Kim, Osborn, and Sensier (2005) possible nonlinearity in the aggregate demand curve might cause the monetary policy rule to be nonlinear. Lastly, Kim, Osborn, and Sensier (2005) an econometric model to specify a flexible nonlinear monetary policy rule to avoid potential misspecification problems of the functions of loss, aggregate supply and demand relations. In the following, we will take Cukierman (2002) model as an example to explain the central bank’s asymmetric behavior of interest rate setting.

Cukierman (2002) proposed three models of overall transmission mechanism, i.e. Lucas, Neo-Keynesian, and New-Keynesian models. It is assumed that exogenous variables are normally distributed. The central bank’s interest rate rules are derived from a loss function, as shown in (1):

\[
L_i = \begin{cases} 
\frac{1}{2}(Ax^2 + \pi_i^2), & \text{when } x_i < 0 \\
\frac{1}{2}(\pi_i^2), & \text{when } x_i \geq 0 
\end{cases},
\]

where \( x_i \) is output gap, i.e. the difference between “real output” and “natural output”, \( \pi_i \) is the difference when inflation deviates from the inflation target. The loss function is asymmetric from the natural output equilibrium, in which the parameter \( A \) is untransparent degree of central bank policy.

Cukierman (2002) pointed out that when \( A = 0 \), the loss function becomes only responsible to inflation target, and the central bank’s behavior becomes clear, that is, only to stabilize price. This is the inflation targeting policy of
New Zealand, Canada, the United Kingdom, and Switzerland described by Geraats (2002). When $A > 0$, the central bank is not only responsible for stabilizing price, but also for stabilizing output. Under this assumption, the central bank has an asymmetric preference. And Cukierman (2002) pointed out that the central bank policy will cause inflation, when the central bank faces more complex goals (which must meet more targets), that is, parameter $A > 0$ and mathematical derivation will find out higher inflation.

2.2. Monetary Policy Transmission Mechanism

Shin (2008) argued how monetary policy rule could affect the economy, while some studies pointed out that the central bank will not affect the economy through interest rate rules. The latter showed that,

1) The central bank only controls short-term interest rate, while what actually affects economic decision-making is long-term interest rates. Blinder (1998, p. 70) pointed out that the Fed controls only the federal funds rate, and has no real effect on the economy.

2) The link between the overnight interest rate and long-term interest rates is through market expectation of future short-term interest rates, that is, expectations of future short-term interest rates will affect long-term interest rates through yield curves.

Empirically, it seems that most studies do not support the policy transmission approach of yield curve. With risk premium adjustment, the yield rate of long-term bonds should be higher. The yield curve should be gradually steeper; in fact, a gradual flat pattern of yield curve is more common. Whether the central bank transmits its policies through yield curve is uncertain. It is more likely that the determination of long-term interest rates is just through market expectations.

As shown by Adrian and Shin (2008a), the central bank could affect long-term interest rate of capital market through the other process to market expectations, and in turn affects consumption and investments of an
They pointed out that through balance sheet management of financial intermediaries, they leverage of financial intermediaries is high during boom of prosperity, and the leverage is low during slumps. During the boom, in order to maintain same proportion or higher financial leverage, the scale of balance sheet is easily expandable, and banks need to purchase more assets. By contrast, during the slump in aggregate demand, banks must sell assets. So the balance sheet management (or called leverage management) behavior of financial institutions will exacerbate along with business cycle (pro-cyclical). The boom of the US economy before 2007 caused banks to buy assets, since under the asset price boom in the housing market, banks held lots of liquidity and could not find customers to borrow. Many banks were forced to enter into sub-prime mortgage market, ignoring consumers’ loan repayment ability. Shin (2008) further pointed out that in the real world money supply definition could be extended further. This based on the following reasons:

1) Many financial institutions operating through leveraged business model do not classified in “depository institutions” of traditional definition. The financial intermediation role is no longer solely dominated by banks due to popularity of asset securitization. The “shadow banking system”, composed of hedge funds, investment banks, and other non-depository financial firms, provides low interest-rate funds to consumers in the mortgage and auto loan markets. These loans are funded primarily by repurchase agreements (repos), using assets like mortgage-backed securities (MBS) and asset-backed securities (ABS) as collateral (see Gorton and Metrick (2012)).

2) Banks’ liabilities are not composed by deposits alone. Banks can obtain funds from financial markets. Base on observation, Shin (2008) found that the scale of bank’s liabilities varied over time, so the broad definition of money supply was unable to explain the financial cycle of banks.

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1) Financial intermediaries are fund providers and demanders, including banks, mutual funds, hedge funds and investment banks.
Therefore, for the market-oriented financial system (such as in the US), Shin (2008) argued that the best indicator for observing and measuring the financial cycle and liquidity of financial intermediaries is either the scale of collateralized borrowing or repo (repurchase agreements). But the scale of repos is much better. When the short-term interest rate is lower than the one implied by Taylor rule, it could be found that repos rapidly grow, and market liquidity increases, and vice versa. Therefore, the difference between the Taylor-Rule interest rate and actual short-term interest rate can be regarded as discretionary monetary policy. The actual data of observation of Adrian and Shin (2008b) also fitted this inference. Therefore, even through expectations Theory of the term structure could not explain the transmission of monetary policy, Shin (2008) argued that the central bank still has a major influence power on the expectations of long-term interest rate in the market.

For the effect of monetary policy, Shin (2008) proposed that it is still reasonable that the transmission of the central bank policy to financial system is through this balance sheet (or leverage) management approach, rather than through the term structure of future short-term interest rate of expectations theory.

Another case is that when interest rates are at or near zero, the central banks could dramatically increase the supply of base money without changing the policy interest rate. The central banks need non-interest-rate tools, known as nonconventional monetary policy tools, which take three forms: liquidity provision, asset purchases, and commitment to future monetary policy actions (see Mishkin and Eakins (2015, pp. 264-269)). The asset purchase programs have been given the name “quantitative easing” (QE). The substantial increases in base money created by the central banks of the US and UK in 2008 and 2009 in their QE programs was to bring down a range of other interest rates besides the policy interest rate, when the economies experienced a full-scale financial crisis like the one we had recently experienced in 2007-2009 (see Miles, Scott, and Breedon (2012, pp.

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2) In the US, it is the Fed. In this paper, the central bank will be used as a collective name for the policy implementation organization.
339-340). In the US, the Fed purchased government bonds and debts issued by government agencies or government-backed corporation during 2008-2012.

3. THE MODEL FRAMEWORK

In this paper, the model is set to meet the dynamic relationships between various parts, and can be divided into three themes: (1) the behavior of the central bank; (2) the central bank information transmission (see Eijffinger and Shen (2008) and Blinder et al. (2007)); and (3) the overall model setting of a small open economy (see Gali and Monacelli (2005) and Monacelli (2005)). The information transmission process in this study could be expressed in figure 1:

Figure 1  The Information Transmission Process

![Diagram of Information Transmission Process](image-url)
3.1. The Behavior of the Central Bank

The central bank’s reaction equations have been appropriately adjusted and the central bank’s interest rate rule could be classified into four kinds of behaviors:

3.1.1. Inflation targeting (passive monetary policy)

The central bank is only responsible for price inflation deviation from equilibrium, i.e. the central bank needs only guide market price to equilibrium state. For example, the central banks of the UK, Canada, New Zealand, Australia, and Switzerland are near this type of strict inflation targeting central banks. The interest rate rule can be expressed as:

\[ \bar{r}_t = \rho_r \bar{r}_{t-1} + (1 - \rho_r)(\rho_{\pi\rho} \tilde{\pi}_{H,t}), \]

where \( \tilde{\pi}_{H,t} \) represents domestically generated price deviation from equilibrium, \( \bar{r}_t \) represents interest rate response made by central bank to price deviation, \( \rho_{\pi\rho} \) represents response relationship between price and interest rate. For example, when price deviation is 1%, if \( \rho_{\pi\rho} = 1.5 \), central bank will raise interest rate by 1.5%.

3.1.2. The Taylor rule (active and discretionary monetary policy)

The central bank is responsible for both output and price deviation from equilibrium. This rule needs to refer to various economic variables, prone to cause decision-making trade-off, and interest rate behavior can not be clearly expressed. Due to need to consider other economic variables, central bank’s decision-making behavior for controlling price becomes discretionional. Decision-making process is discretionarial when the monetary policy committee members assess whether the current target for the overnight interest rate remains appropriate for achieving the twin goals of low inflation and full employment. A rule of thumb suggested by John Taylor (1993)
builds on the belief held by many economists that the central banks are willing to tolerate a small positive rate of inflation if doing so will help the economy to produce at potential output. Svensson (1997) set an optimization model of the central bank to derive the Taylor rule. Later on, Svensson (2002) presented a monetary policy framework known as flexible inflation targeting. The name reflected the primacy of inflation as the ultimate objective of monetary policy, while the flexibility reflected the short-run tradeoff between inflation control and real economic stability (see Walsh (2009)). The interest rate rule can be expressed as:

\[
\tilde{r}_t = \rho_r \tilde{r}_{t-1} + (1 - \rho_r)(\rho_x x_t + \rho_{r\pi} \tilde{\pi}_{H,t}),
\]

where \( x_t \) represents output gap, \( \rho_x \) represents proportional relationship between output gap and interest rate conduct.

### 3.1.3. Exchange-rate targeting

Under uncovered interest parity (UIP) hypothesis, pegged exchange rate will cause the central bank lose interest rate pricing right, e.g. Hong Kong:

\[
\tilde{r}_t - \tilde{r}^*_t = E[\Delta e_{t+1}],
\]

under the condition of \( \tilde{e}_t = 0 \), \( \tilde{r}_t = \tilde{r}^*_t \), where \( \tilde{r}^*_t \) represents change in interest rate of foreign countries, and \( \Delta e_{t+1} \) is expected exchange rate change in next period. Equation (4) represents that domestic interest rate will change equally with foreign interest rate.

### 3.1.4. Hybrid or asymmetric inflation targeting

The central bank’s behaviors on two sides of equilibrium state are not the same. For example, the ECB in the EURO zone. Dolado, Maria-Dolores, and Naveria (2005) found empirical support for this type of asymmetries in the interest rate-setting behavior of four European central banks, i.e.
Germany, France, Spain, and Euro area, during 1980-2001 approximately. The following two cases exist according to Cukierman (2002):

Case 1
When economy is overheating, actual output is higher than equilibrium output. The central bank is less sensitive to output gap, and needs only to control price inflation.

Case 2
When economy is in recession, output gap is less than zero. Under various pressures, the central bank will try to avoid output crunch and prevent unemployment expansion. At this point, the central bank will be very sensitive to negative deviation of output (i.e., output gap is less than 0). In addition to controlling price, at this time, the interest rate rule of central bank will consider to stimulate output.

The central bank will have different reactions when facing with these two economic situations. This is an asymmetric behavior of the central bank. In addition to affecting central bank’s behavior, the positive and negative characteristics of output gap will also make the central bank have different reaction equations. Compared to the previous rule setting, it is more difficult to grasp central bank’s decision, since it adopted this asymmetric policy and its behavior is not symmetric. Thus, the interest rate rule can be expressed as:

\[
\tilde{r}_t = \rho_r \tilde{r}_{t-1} + (1 - \rho_r) \left( \frac{\rho_{x_t} \tilde{x}_{H,t}}{\rho_{r_t} x_t + \rho_{d_t} \tilde{\pi}_{H,t}} \right)
\]

where \( x_t \geq 0 \) (when boom) and \( x_t < 0 \) (when slump),

\[
\tilde{r}_t = \rho_r \tilde{r}_{t-1} + (1 - \rho_r) \left( \frac{\rho_{x_t} \tilde{x}_{H,t}}{\rho_{r_t} x_t + \rho_{d_t} \tilde{\pi}_{H,t}} \right)
\]

where the definitions of variables are same as the previous text.
3.2. Setting of Central Bank Information Transmission

As pointed out by Adrian and Shin (2008), and Shin (2008), although the expectations theory of the term structure fails to explain differences in bond interest rates across different maturities, the transmission of central bank monetary policy to financial system is through balance sheet (or leverage) management of financial intermediaries. That is, the central bank could affect interest rates of all maturities. And it could control not only the overnight interest rate, but also market liquidity due to discretionary monetary policy, such as QE to purchase bonds and liquidity provision to non-depository financial institutions.

In addition, it should be noted that although most studies do not support the policy transmission approach of yield curve, it is still reasonable to explain the market behavior of interest rates through the expectations theory of the term structure of interest rates. Thus, in the following, we will follow the existing literature of the term structure of interest rates theory to set up the market behavior of interest rates.

Eijffinger and Shen (2008) and Blinder et al. (2007) considered that long-term interest rate affecting consumption decisions can be composed by a combination of a series of short-term interest rates:

\[
(1 + LR)^e = [(1 + r_t)(1 + r_{t+1}^e)(1 + r_{t+2}^e)\cdots(1 + r_{t+n-1}^e)],
\]

where \( LR_t \) is long-term interest rate, \( r_t \) is today’s (time \( t \)) overnight interest rate; \( r_{t+1}^e \) is expected overnight interest rate of time \( t+1 \).

Due to log-linearization in the model, the interest rate term structure is converted to the varied form in equation (7):

\[
LR_t = \frac{1}{n} (\bar{r}_t + \bar{r}_{t+1}^e + \bar{r}_{t+2}^e + \cdots + \bar{r}_{t+n-1}^e),
\]
where \( lr \) is the long-term interest rate deviation from equilibrium, \( \tilde{r}_{t+1}^e \) represents the degree of short-term interest rate deviation from equilibrium expected by the public at time \( t+1 \), which is formed by a function. In order to simplify the model, the short-term interest rate of period \( t+1 \) expected by the public is determined by price and output observed at period \( t \) (present). At the same time the central bank can release interest rate information \( s_{t+1} \) of period \( t+1 \). That is, the central bank announces information about short-term interest rate of period \( t+1 \) to the public, therefore. Thus \( \tilde{r}_{t+1}^e \) can be expressed as equation (8):

\[
\tilde{r}_{t+1}^e = (1 - \rho_{x_t, t+1}) (\rho_{x_{t+1}, t+1}\tilde{r}_{t+1} + \rho_{x_{t+1}, t+1} x_t) + \rho_{x_{t+1}, t+1} s_{t+1},
\]

where \( \rho_{x_t, t+1}, \rho_{x_{t+1}, t+1} \) represent forecasting parameters of the public for short-term interest rate of period \( t+1 \) after observing current economic information, while \( \rho_{x_{t+1}, t+1} \) represents central bank’s information transparency. For information flowed out of the central bank, we assume the information is short-term interest rate of period \( t+1 \), i.e. \( s_{t+1} = \tilde{r}_{t+1} \). Thus, equation (8) can be expressed as equation (9):

\[
\tilde{r}_{t+1}^e = (1 - \rho_{x_t, t+1}) (\rho_{x_{t+1}, t+1}\tilde{r}_{t+1} + \rho_{x_{t+1}, t+1} x_t) + \rho_{x_{t+1}, t+1} \tilde{r}_{t+1},
\]

when \( \rho_{x_{t+1}, t+1} = 1 \), equation (9) becomes \( \tilde{r}_{t+1}^e = \tilde{r}_{t+1} \), that is, the public can predict future short-term interest rate fully based on central bank’s information. At this time the central bank’s policy is very transparent. If \( \rho_{x_{t+1}, t+1} = 0 \), then equation (9) becomes \( \tilde{r}_{t+1}^e = \rho_{x_{t+1}, t+1}\tilde{r}_{t+1} + \rho_{x_{t+1}, t+1} x_t \). At this time the public must collect current data by their own in order to predict short-term interest rate of period \( t+1 \). So the central bank’s policy information is very untransparent. Therefore, the expected short-term interest rate of all the other periods can be expressed as:
\[ r_{t+1}^e = (1 - \rho_{t+1}^e)(\rho_{t+1}^e \tilde{r}_{t+1} + \rho_{t+1}^e x_t) + \rho_{t+1}^e \tilde{r}_{t+1}, \]
\[ r_{t+2}^e = (1 - \rho_{t+2}^e)(\rho_{t+2}^e \tilde{r}_{t+2} + \rho_{t+2}^e x_t) + \rho_{t+2}^e \tilde{r}_{t+2}, \]
\[ \vdots \]
\[ r_{t+n}^e = (1 - \rho_{t+n}^e)(\rho_{t+n}^e \tilde{r}_{t+n} + \rho_{t+n}^e x_t) + \rho_{t+n}^e \tilde{r}_{t+n-1}. \]

In \( \tilde{r}_{t+1}^e, \tilde{r}_{t+2}^e, \ldots, \tilde{r}_{t+n-1}^e \), the coefficients of \( \tilde{r}_{t,l} \) and \( x_t \) are the combination of equations of \( \rho_{t+1}^e \) and \( \rho_{t+n}^e \) in different periods, which could be assumed to be constant to simplify parameter estimation. The equation parameters can be expressed as: \( \rho_{t}^e \) and \( \rho_{n}^e \). Thus, we could obtain (10):

\[ r_{t+1} + r_{t+2} + \ldots + r_{t+n-1}^e = (1 - \rho_{t}^e)(\rho_{t}^e \tilde{r}_{t} + \rho_{n}^e x_t) + \rho_{t}^e (\tilde{r}_{t+1} + \ldots + \tilde{r}_{t+n-1}). \]  

(10)

under computer simulation, since the aim of this paper is to show the central bank’s interest rate term structure, it will simplify the analysis of the problem by setting \( n = 4 \). That is, the determination of long-term interest rate contains interest rate term structure of 4 periods (for example, 4 quarters or years). Thus, equation (11) can be sorted out as:

\[ l_{t} = \frac{1}{n}(\tilde{r}_{t} + (1 - \rho_{t}^e)(\rho_{t}^e \tilde{r}_{t} + \rho_{n}^e x_t) + \rho_{t}^e (\tilde{r}_{t+1} + \ldots + \tilde{r}_{t+n-1})), \]

(11)

If the central bank carries out high transparent policy, then \( \rho_{t} \) closes to 1. Equation (12) becomes:

\[ l_{t} = \frac{1}{4}(\tilde{r}_{t} + \frac{3}{4}(1 - \rho_{t}^e)(\rho_{t}^e \tilde{r}_{t} + \rho_{n}^e x_t) + \frac{1}{4}\rho_{t}^e (\tilde{r}_{t+1} + \tilde{r}_{t+2} + \tilde{r}_{t+3})). \]

(12)

If \( \rho_{t} \) closes to 0, then the public have to guess central bank’s future short-term interest rate by their own according to current period economic situation.

3) To set \( n = 4 \) is only for simplify the analysis. This is to show the term structure of interest rates so that there is difference between short and long term interest rates. It is straightforward to extend the period to any longer periods, say \( n = 10 \).
3.3. Dynamic General Equilibrium System of a Small Open Economy

The model setting follows a small open economic model established by Gali and Monacelli (2005) and Monacelli (2005). The model derived by Gali and Monacelli (2005) was extended from a closed economy model of Gali and Gertler (1999). Later Monacelli (2005) further generalized this model in order to meet the requirements of purchasing power parity (PPP) assumption. In this model, the foreign economy is composed of many small countries, and domestic economy also is a small economy, so it can not affect foreign economic environment. Thus, foreign economic variables are not affected by the changes in domestic economy. That is, foreign conditions belong to exogenous shocks. We assumed that financial market is perfect, no arbitrage, meeting the hypothesis of interest rate parity condition and joint risk sharing.

The present study followed Gali and Gertler (1999) and Matheson (2006) to modify firms’ behavior of expectation to be able to carry out forward-looking and backward-looking. Within the model, part of firms will predict next period price inflation to carry out price adjustment, while another part of firms can only carry out price adjustment based on observed price inflation of current period. And both the two types of firms meet Calvo-type pricing, that is, there exists a certain degree of price stickiness. This assumption meet Neo-Keynesian and New-Keynesian price sluggish assumptions.

3.3.1. The household

Assuming the objective of the representative individual is to pursue lifetime utility maximization under budget constraint:

$$\max \sum_{t=0}^{\infty} \beta^t U(C_t, N_t),$$

where $\beta$ is the discount factor; $C$ is current consumption and generate a positive utility; $N$ is current period work effort and generate a negative utility;
while the single-period utility function can be expressed in constant elasticity of substitution (CES) form:

$$ U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \frac{N_t^{1+\phi}}{1+\phi}, $$

(14)

where $\sigma$ represents the inverse elasticity of consumption substitution, $\phi$ represents the inverse of elasticity of labor supply substitution.\(^4\) The intertemporal budget constraint is:

$$ p_t C_t + e_t[Q_{t,t+1}D_{t+1}] \leq W_t N_t + D_t + T_t, $$

(15)

where $p_t$ is current period price, $D_t$ is expected earning of financial assets held by the public. The discount rate is long-term interest rate rather than the short-term interest rate of central bank because the duration of net financial assets held by the public will exceed one period, and will not be settled every period. $W_t$ is wage, $T_t$ is net tax and transfer income, $Q_{t,t+1}$ is present value of financial assets held at period $t+1$ with respect to period $t$, and $e_t$ is expectation operator at period $t$. In this model, assuming that financial market is perfect, so after exchange rate conversion to domestic currency a representative individual will have same reward no matter where he invest in any financial asset. Under this assumption, interest rate parity hypothesis and risk value equal principle can be met (so no risk premium between domestic and foreign interest rates). Thus, the foreign interest rate can fully affect domestic financial market through this link.

The household problem can be solved to get:

Current period choice: 

$$ C_t^\sigma N_t^\phi = \frac{W_t}{P_t}, $$

(16)

Intertemporal choice: 

$$ Q_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right), $$

(17)

\(^4\) Benefit of adopting constant elasticity of substitution (CES) expression method is that after log-linearization the expression form would be simpler.
where \( Q \) is the inverse of long-term risk-free deposits gross rate:

\[
\frac{1}{Q_{t, t+1}} = LR_t = 1 + lr_t, \quad \text{and (17) becomes:}
\]

\[
\beta LR_t[(\frac{C_{t+1}}{C_t})^{\sigma}(\frac{P_t}{P_{t+1}})] = 1,
\]

(18)

where \( LR_t \) is expected total value of each one dollar of long-term asset at period \( t \) after held for one period; \( lr_t \) is the long-term yield rate. The interest rate controlled by the central bank can not directly affect \( lr_t \), but it affects long-term rate through interest rate term structure of equation (12).

To facilitate dynamic research, log-linearizing (16) and (18) around the equilibrium values:

\[
\tilde{\bar{w}}_t - \tilde{\bar{p}}_t = \sigma \tilde{c}_t + \phi \tilde{n}_t,
\]

(19)

\[
\tilde{c}_t = E_t \tilde{c}_{t+1} - \frac{1}{\sigma} [lr_t - E_t \tilde{n}_{t+1}],
\]

(20)

where equation (20) is the setting for meeting central bank information transparency in section 2, so the household’s consumption decision is not directly controlled by the central bank’s short-term interest rate, but determined by the short-term interest rate of current period and the future short-term interest rate predicted through the observation of current economic parameters, i.e. a long-term interest rate. The household still needs to make decision every period although it takes the long-term rate as the discount rate, so modify equation (20) as:

\[
\tilde{c}_t = E_t \tilde{c}_{t+1} - \frac{1}{\sigma} [lr_t - E_t \tilde{n}_{t+1}],
\]

(21)

where \( LR_t \) represents annualized expected long-term interest rate.

---

5) Log-linearization is a percentage expression method, expressing degree of deviation from equilibrium state of a variable.

3.3.2. Further discussions of the household

In an open economy, consumption goods are both imported and produced domestically. The degree of openness is $\alpha$, that is, the economy has $\alpha$ ratio of international trade. The composition of goods $\theta > 0$ represents elasticity of substitution between domestic and imported products, expressed in constant elasticity of substitution (CES) form:

$$C_t = [(1 - \alpha)\theta^\eta + \alpha^\eta]^{1/\eta},$$

(22)

where $\eta$ represents elasticity of price substitution between domestic and foreign goods; $C_{ht}$ represents consumption of domestically produced product; $C_F$ represents consumption of imported goods. Where $C_{ht,t}$ is composed of many domestic products, index $j$ represents various small countries:

$$C_{ht,t} = \int_0^1 C_{ht,t}(j) \frac{\varepsilon-1}{\varepsilon} dj\frac{\varepsilon}{\varepsilon-1},$$

(23)

where $\varepsilon$ represents elasticity of substitution between domestic consumer goods; $C_{Ft,t}$ represents consumption of imported goods; composed of many domestic goods; index $j$ represents various small countries:

$$C_{Ft,t} = \int_0^1 C_{Ft,t}(j) \frac{\gamma-1}{\gamma} dj\frac{\gamma}{\gamma-1},$$

(24)

where $\gamma$ represents elasticity of consumption substitution between foreign consumer goods. The behavior of foreign consumer is assumed the same as that of domestic consumer.

Integrate consumption $C_{it,t}$ over various countries, can derive domestic consumption for foreign goods:

$$C_{Ft,t} = \int_0^1 C_{Ft,t}(j) \frac{\gamma-1}{\gamma} dj\frac{\gamma}{\gamma-1},$$

(25)

where $\gamma$ represents elasticity of consumption substitution between foreign
countries. Thus, the budget constraint equation can be rewritten as:

\[ \int_0^1 P_{H,t}(j)C_{H,t}(j) dj + \int_0^1 P_{F,t}(j)C_{F,t}(j) dj + E_t[Q_{t+1}, D_{t+1}] \leq D_t + W_tN_t + T_t, \]

where \( P_{H,t}(j) \) represents price of domestic goods; \( P_{F,t}(j) \) represents price of a foreign goods.

Bring the above composition of consumption into budget constraint equation, the optimal demand curve of each goods becomes:

\[
C_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{1-\varepsilon} C_{H,t}, \quad (26)
\]

\[
C_{F,t}(j) = \left( \frac{P_{F,t}(j)}{P_{F,t}} \right)^{1-\varepsilon} C_{F,t}. \quad (27)
\]

Let price is also expressed in constant elasticity of substitution (CES) form:

\[
P_{H,t} = \int_0^1 P_{H,t}(j) \left( \frac{1}{j^{1-\varepsilon}} \right) dj^{1+\varepsilon},
\]

\[
P_{F,t} = \int_0^1 P_{F,t}(j) \left( \frac{1}{j^{1-\varepsilon}} \right) dj^{1+\varepsilon}.
\]

The aggregate consumptions and aggregate prices can be expressed as:

\[
\int_0^1 P_{H,t}(j)C_{H,t}(j) dj = P_{H,t}C_{H,t},
\]

\[
\int_0^1 P_{F,t}(j)C_{F,t}(j) dj = P_{F,t}C_{F,t}.
\]

From equations (24), (25), we can express aggregate consumption goods as:
\[ C_{H,t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \]  
(28)

\[ C_{F,t} = \alpha \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t. \]  
(29)

The general price level expressed in equation (30) still comply with constant elasticity of substitution (CES) form to express

\[ P_t = \left[ (1 - \alpha) \bar{p}_{H,t}^{1-\eta} + \alpha \bar{p}_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \]  
(30)

As for log-linearized expression, after taking logarithm of (30) and subtracting from the equilibrium value, the general price level can be expressed as:

\[ \tilde{p}_t = (1 - \alpha) \tilde{p}_{H,t} + \alpha \tilde{p}_{F,t}, \]  
(31)

where \( \tilde{p}_t \) represents log-linearized expression of consumer price index; \( \tilde{p}_{H,t} \) represents log-linearized expression of domestic goods price; \( \tilde{p}_{F,t} \) represents log-linearized expression of imported goods price.

### 3.3.3. The firm

The products produced by firms are available for domestic use and export with a linear production function:

\[ Y_i(i) = A_i N_i(i), \]  
(32)

where \( A_i \) represents technology, \( N_i(i) \) represents labor input. After integration, we have aggregate production function:

\[ Y_t = A_t N_t, \]  
(33)
In log-linearized form:

\[ \tilde{y}_t = \tilde{a}_t + \tilde{n}_t, \]  

(34)

where the lowercase represent corresponding log-linearized expressions.

Given output, the representative firm minimizes its cost (total cost) under the restriction of (33). The production factor is provided by domestic labor. Taking domestic price as denominator, the lowest cost of firm can be expressed as:

\[ \min \frac{W}{P_{H,t}} N_t, \]  

(35)

where \( W \) represents domestic wage.

Substituting equation (33) into equation (35) and differentiate to solve firm’s problem:

\[ MC_t = \frac{W_t}{P_{H,t} A_t}, \]  

(36)

where \( MC \) represents firm’s marginal cost. Taking log-linearization and we have:

\[ m\tilde{c}_t = \tilde{\tilde{w}}_t - \tilde{P}_{H,t} - \tilde{a}_t, \]  

(37)

where the lowercases represent their corresponding log-linearized expressions.

3.3.4. Price setting of firms

Follow the sticky price model of Gali and Monacelli (2005), Calvo (1983), Yun (1996), and McCandless (2008), we constructed a hybrid Philips Curve in accordance with method of Gali and Gertler (2007) and Matheson (2006). In the economy firm’s price is sticky. Each period only \( \theta_h \) part of firms are unable to adjust price; however, the remaining \( \theta_{(1-h)} \) part of firms are
able to adjust price. The price of current period is expressed as:

\[ \tilde{p}_{H,t} = \theta_H \tilde{p}_{H,t-1} + (1 - \theta_H) \bar{p}_{H,t}, \]

(38)

where \( \theta_H \) represents Calvo parameter, and \( \tilde{p}_{H,t-1} \) represents pricing expected by firms.

Following the derivation in Appendix of Gali and Monacelli (2005), a forward-looking firm will generate the pricing behavior (expressed in log-linearized form):

\[ \tilde{p}^F_{H,t} - \tilde{p}_{H,t-1} = (1 - \beta \theta_H) \sum_{k=0}^{\infty} (\beta \theta_H)^k E_t[m c_{t+k} + \tilde{p}_{H,t+k} - \tilde{p}_{H,t-1}] \]

(39)

where \( \tilde{p}^F_{H,t} \) represents current period price-setting of forward-looking firms. And \( p_{H,t+k} - p_{H,t-1} \) can be expressed as:

\[ \tilde{p}_{H,t+k} - \tilde{p}_{H,t-1} = (\tilde{p}_{H,t} - \tilde{p}_{H,t-1}) + (\tilde{p}_{H,t+1} - \tilde{p}_{H,t}) \]

\[ + (\tilde{p}_{H,t+2} - \tilde{p}_{H,t+1}) + \ldots = \sum_{k=0}^{\infty} \tilde{p}_{H,t+k}, \]

(40)

after sorting out, we have:

\[ \tilde{p}^F_{H,t} - \tilde{p}_{H,t-1} = (1 - \beta \theta_H) \sum_{k=0}^{\infty} (\beta \theta_H)^k E_t[m c_{t+k} + \sum_{k=0}^{\infty} \tilde{p}_{H,t+k}] \]

(41)

where \( m c_{t+k} \) is the deviation of marginal cost from natural condition, and under perfect competitive situation, it is zero when equilibrium.

The expression of next period becomes:

\[ \tilde{p}^F_{H,t+1} - \tilde{p}_{H,t} = (1 - \beta \theta_H) \sum_{k=1}^{\infty} (\beta \theta_H)^k E_t[m c_{t+k} + \sum_{k=0}^{\infty} \tilde{p}_{H,t+k}] \]

(42)

Substituting (42) into (41), it becomes:

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Inflation Targeting, Exchange-Rate Targeting Monetary Policies: A DSGE Model

\[ \tilde{P}_{H,t}^F - \tilde{P}_{H,t-1}^F = (1 - \beta \theta_H) m\tilde{c}_t + \tilde{\pi}_{H,t}^F + \beta \theta_H E_1 [\tilde{P}_{H,t+1}^F - \tilde{P}_{H,t}^F], \tag{43} \]

the forward-looking price inflation and price stickiness has a fixed ratio relationship: \( \tilde{\pi}_{H,t}^F = (1 - \theta_H)(\tilde{P}_{H,t}^F - \tilde{P}_{H,t-1}^F) \), so equation (43) can be solved:

\[ \tilde{\pi}_{H,t}^F = \beta E_1 [\tilde{\pi}_{H,t+1}^F] + \lambda m\tilde{c}_t, \tag{44} \]

where \( \lambda_H = \frac{(1 - \theta_H)(1 - \beta \theta_H)}{\theta_H} \).

As for a backward-looking firms, its pricing behavior is adjusted in accordance with price inflation observed in previous period:

\[ \tilde{P}_{H,t}^B = \tilde{P}_{H,t-1}^B + \tilde{\pi}_{H,t}^B, \tag{45} \]

where \( \tilde{P}_{H,t}^B \) is current period pricing of backward-looking manufacturers, \( \tilde{P}_{H,t-1}^B \) is previous period optimal pricing of backward-looking manufacturers.

Therefore, after mixed these two types of firms (forward-looking firms accounts for the proportion of \( w_H \), and backward-looking firms accounts for the remaining proportion of \( 1 - w_H \)), the adjustable new price can be expressed:

\[ \tilde{P}_{H,t} = (1 - w_H)\tilde{P}_{H,t}^F + w_H \tilde{P}_{H,t}^B. \tag{46} \]

In equilibrium, \( m\tilde{c} \) represents its deviation degree when subject to exogenous effect. After sorting out and using the way of subtraction between two periods, we have a hybrid Phillips curve:

\[ \tilde{P}_{H,t+1} - \tilde{P}_{H,t} = \gamma_{H,F} E_1 [\tilde{\pi}_{H,t+1}] + \gamma_{H,B} [\tilde{\pi}_{H,t+1}] + \lambda_H m\tilde{c}_t, \tag{47} \]

where \( \phi_H = \theta_H + w_H (1 - \theta_H (1 - \beta)) \); \( \lambda_H = \frac{(1 - w_H)(1 - \theta_H)(1 - \beta \theta_H)}{\phi_H} \);
Parameter definitions above have been shown before.

\( m\tilde{c}_i = mc - mc \) is degree of deviation from equilibrium state of marginal cost, and \( mc = \log \frac{1}{1 - \mu} = -\mu \) is the mark-up to consumer demand elasticity of monopolistic competitors. To simplify parameters, we assume market competition degree will not change over time, \( mc \) will close to a fixed constant to meet same market competition degree. So set \( m\tilde{c}_i = 0 \) in equilibrium, we can derive the marginal cost of natural output.

In this model, \( \theta_H \) is the degree of price stickiness, the closer the \( \theta_H \) to 1, the more the stickiness of price. And \( w_H \) is the degree of backward-looking. As \( w_H \) is closer to 0, firm’s behavior is closer to a backward-looking Neo-Keynesian economy; while when \( w_H \) is closer to 1, the firm is forward-looking, and its behavior is closer to a New-Keynesian economy.

### 3.3.5. Price setting of imported goods

In this section, we follow Calvo’s price setting, and assume \( \theta_F \) part is sticky. Like previous section, having a hybrid backward-looking and forward-looking, with \( w_F \) and \( (1 - w_F) \) respectively, thus

\[
\tilde{p}_{F,i} = \theta_F \tilde{p}_{F,i-1} + (1 - \theta_F) \tilde{p}_{F,i-1},
\]

where \( \tilde{p}_F \) represents import price, \( \bar{p}_F \) represents firm’s optimal price of import. In the part of price-setting forward-looking directly referenced result of Monacelli (2005).\(^8\)

\[
\tilde{p}_{F,i}^F = (1 - \beta \theta_F) \sum_{k=0}^{\infty} (\beta \theta_F)^k \{ \psi_{F,i+k} + \tilde{p}_{F,i+k} \},
\]

where \( \tilde{p}_{F,i}^F \) represents current period price setting of forward-looking

---

\(^8\) Monacelli (2005).
importers, $\Psi_t$ represents ppp gap, $\psi_t$ is its logarithmic expression of deviation from equilibrium.

For backward-looking firms, current period pricing can be expressed as:

$$\tilde{p}^b_{F,t} = p_{F,t-1} + \tilde{r}_{F,t-1},$$

(50)

where $\tilde{p}^b_F$ represents current period setting of backward-looking. After weighted it, the final new price can be expressed as:

$$p_{F,t} = (1 - w_F) \tilde{p}^b_{F,t} + w_F \tilde{p}_F^e,$$

(51)

Considering (48) to (51), we can derive Hybrid Philips Curve:

$$\tilde{p}_{F,t} = \gamma_{F,F} E_t [\tilde{p}_{F,t+1}] + \gamma_{B,F} E_t [\tilde{r}_{F,t+1}] + \lambda_F \psi_{F,t},$$

(52)

where $\phi_F = \theta_F + w_F (1 - \theta_F (1 - \beta)); \lambda_F = \frac{(1 - w_F)(1 - \theta_F)(1 - \beta \theta_F)}{\phi_F}$; $\gamma_{F,F} = \frac{\theta_F}{\phi_F}; \gamma_{B,F} = \frac{w_F}{\phi_F},$ and parameter definitions are shown before.

### 3.3.6. Exchange rate, terms of trade, and non-complete pass-through

In Gali and Monacelli (2005), a small open economy is assumed to satisfy PPP. Later the PPP hypothesis is relaxed, and Monacelli (2005) revised to introduce a new variable, i.e. purchasing power parity gap (PPP gap, $\Psi$, log-linearized expression is $\psi$), so that imported goods will also be affected by import price in addition to being affected by foreign price inflation. If PPP gap is equal to 0, then the import price in previous section is the product of foreign aggregate price and exchange rate, as shown in Gali and Monacelli (2005).

Terms of trade ($S_t$, expressed in log-linearized form: $s_t$) can be expressed as:

$$S_t = \frac{P_{F,t}}{P_{F,t}} = \int_0^1 S_{t+1} d\frac{1}{\sqrt{r}},$$

(53)
where index $i$ represents $i$th foreign economy, and integrating in CES form to get aggregate terms of trade. Terms of trade is import price/local production price. Under this definition, if import price is higher, local production price is relatively lower, it will be favorable to export. $S_i$ will rise. Express logarithm approximation around the equilibrium state:

$$\tilde{s}_i = \tilde{p}_{F,i} - \tilde{p}_{H,i};$$ (54)

Substituting (46) and (51) into the aggregation of prices, and we get:

$$\tilde{p}_i = (1 - \alpha)\tilde{p}_{H,i} + \alpha \tilde{p}_{F,i} = \tilde{p}_{H,i} + \alpha \tilde{s}_i,$$ (55)

after intertemporal subtraction, we have:

$$\delta \tilde{p}_i = \delta \tilde{p}_{H,i} + \alpha \Delta \tilde{s}_i.$$ (56)

The real exchange rate: $Q = \frac{EX_i P_i^*}{P_i}$, where $EX_i$ represents nominal exchange rate, and $Q_i$ represents real exchange rate. Expressed in log-linear approximation:

$$\tilde{q}_i = \tilde{e}_i + \tilde{p}_i^* - \tilde{p}_i = \psi_{F,i} + (1 - \alpha)\tilde{s}_i,$$ (57)

where $q_i$ and $e_i$ represents logarithmic values of real and nominal exchange rates, $p_i^*$ is foreign price. The purchasing power parity gap (PPP gap, Law-of-one price gap) can be expressed as:

$$\psi_{F,i} = \tilde{e}_i + \tilde{p}_i^* - \tilde{p}_{F,i}.$$ (58)

If set $\psi_{F,i} = 0$, then it is back to the setting of Gali and Monacelli (2005).
3.3.7. Joint risk-sharing, interest rate parity, and substantiation of marginal cost

1) Joint risk-sharing

In theory, financial markets around the world should have same return as long as investment instrument is the same. Thus, a foreign investor has same intertemporal condition like (17) (and considering exchange rate):

\[
\beta \left( \frac{C_{i,t+1}}{C_t} \right)^{\sigma} \left( \frac{P_{t+1}}{P_{t}} \right) = \beta \left( \frac{C_{i,t+1}^*}{C_t^*} \right)^{\sigma} \left( \frac{P_{t+1}^*}{P_{t}^*} \right) \left( \frac{EX_{t+1}}{EX_{t}} \right) = X_{t,t+1},
\]

where \( C^* \) represents foreign consumption. By considering the real exchange rate definition, (59) can be rewritten as:

\[
\left( \frac{C_{i,t+1}}{C_t} \right) = \left( \frac{C_{i,t+1}^*}{C_t^*} \right) \left( \frac{Q_{t+1}}{Q_{t}} \right)^{\sigma}.
\]

In addition, assume that \( C_t = \nu C_t^* Q_t^{\sigma} \), that is, domestic consumption and foreign consumption has a specific proportional relationship (see Appendix A, Gali and Monacelli (2005)), where \( \nu \) had been fixed during optimization process.

Therefore, intertemporal relationship (59) become a single period consideration: \( C_t = C_t^* Q_t^{\sigma} \). After log-linearization, we have:

\[
\tilde{c}_t = \tilde{c}_t^* + \frac{1}{\sigma} \tilde{q}_t,
\]

substituting \( \tilde{q}_t \) in (57) and we get:

\[
\tilde{c}_t = \tilde{c}_t^* + \frac{1}{\sigma} \left( (1-\alpha) \tilde{s}_t + \psi_{F,t} \right).
\]

2) Interest rate parity

The exchange rate can be linked through interest rate parity condition:

\[
\tilde{r}_t^* - \tilde{r}_t^* = E_t \left[ \Delta e_{t+1} \right],
\]

where \( r_t^* \) is foreign interest rate, \( \Delta e_{t+1} = \tilde{e}_{t+1} - \tilde{e}_t \) is the change in exchange rate.
3) Substantiation of marginal cost

Before carrying out this model, the marginal cost must be substantiated. Thus, after arrangement of (19), (34), (37), (57), (60) and (61), we get substantiated change of marginal cost:

\[ m \ddot{c}_i = \sigma \ddot{y}_i + \phi \ddot{y}_i + \ddot{s}_i - (1 + \phi) \ddot{a}_i. \]  \hspace{1cm} (63)

3.3.8. Market clearing

Under the market clearing condition, all domestically produced consumer goods will be consumed for domestic consumption and export: \( Y_i = C_{H,i} + C_{H,i}^* \). From equation (28), domestic consumption can be expressed in log-linearized form:

\[ \ddot{c}_{H,i} = \eta \ddot{a}_i + \ddot{c}_i. \]  \hspace{1cm} (64)

Imported goods consumption in equation (29) is \( C_{F,i} = \alpha \left( \frac{P_{F,i}}{P_i} \right)^{-q} C_i \), and foreign consumption of domestically produced goods is assumed correspondingly from this equation (symmetric, considering exchange rate conversion): \( C_{H,i}^* = \alpha \left( \frac{P_{H,i}}{EX_i P_i} \right)^{q} C_i^* \). Taking log-linearization and then summing over all countries, we have the demand of export product:

\[ \ddot{c}_{H,i}^* = \eta (\ddot{s}_i + \ddot{\psi}_i) + \ddot{c}_i^*. \]  \hspace{1cm} (65)

Relative to domestic economy, foreign countries can be thought as a whole economy, therefore, \( \ddot{c}_i^* = \ddot{y}_i^* \). Under the market clearing condition (after logarithmic linearization):

\[ \ddot{y}_i (i) = (1 - \alpha) \ddot{c}_{H,i} (i) + \alpha \ddot{c}_{H,i}^* (i). \]

After summation over all countries or all consumed goods, we have:
\( \ddot{y}_t = (1 - \alpha)\ddot{c}_{t-1} + \alpha \ddot{c}_{t-1}^*, \) 

(66)

3.4. Exogenous Shocks

Assume each external shock follows an autocorrelation with one order (AR (1)), and the autocorrelation coefficient is 0.9. These shocks include:

1) Production technology shock \((A)\): impact on technological factor of production function.
2) Foreign income shock \((y^*)\): impact on foreign income.
3) Terms of trade shock \((s)\): impact on terms of trade.
4) Foreign interest rate shock \((r^*)\): impact on foreign interest rate.
5) Foreign price shock \((\pi^*)\): impact on foreign price.
6) Domestic cost side shock \((\pi_H)\): impact on domestic Phillips Curve.

3.5. Parameter Settings

This paper adopted the MATLAB plug-in program Dynare. This is a software specially designed for simulating economic dynamics, it can simulate non-linear or linear model. In addition to MATLAB platform, Dynare is also available on Gauss platform. For relevant use information and user manual, please refer to first page of Dynare (or Dynare’s homepage).\(^9\) All the simulations of this paper use syntax of Dynare, and operate on MATLAB platform.

Dynare has built-in Bayesian estimation method. The use of Bayesian method to estimate DSGE model parameters was first adopted by Schorfheide (2000), and the use in a small open economy was first adopted by Lubik and Schorfheide (2003). The benefit of adopting this method is that it is not necessary to precisely estimate a parameter, but just have to describe possible range and distribution of the parameter in advance, and then appropriate parameter can be estimated through real data and model assumptions. This parameter is not a real parameter, but the most

\(^9\) Homepage of Dynare (available at: http://www.cepremap.cnrs.fr/juillard/mambo/).
appropriate collocation of model and data under the collocation of real data and the given model. Its principle is to estimate posterior probability based on prior probability, and the posterior probability can be assigned as:

$$p(\theta | Y^T) = \frac{L(\theta | Y^T)p(\theta)}{\int L(\theta | Y^T)p(\theta)d\theta},$$  \hspace{1cm} (67)$$

where $p(\theta)$ is prior distribution, and $L(\theta | Y^T)$ is likelihood function of given condition $Y^T$, determined by the one who set the model. It is assumed to be a normal, Beta, or other distributions. So the posterior distribution can be derived from estimation of equation (67).

After choosing prior distribution, and coordinated with model assumptions, posterior distribution can be plotted. The stochastic process uses Monte Carlo Markov Chain to continuously generate posterior probability distribution. This paper used Bayesian estimation method built in Dynare. Dynare derives initial value mainly through maximum likelihood method, and then constantly throw in adjustment parameter through Monte Carlo method, and the parameter distribution of best fit model can be derived through posterior deduction.

This study collected quarterly data of per capita real output of Taiwan from 1982:Q1 to 2007:Q4, and data source is TEJ database. After taking logarithm for the data, then using Hodrick-Prescott (HP) Filter Method to remove trend of time series. This method was used in Hodrick and Prescott (1997), later commonly used by Real Business Cycle (RBC) model to remove the trend. After these processes, only the deviation items after taking the logarithm will remain.

Under the assumptions of prior distribution of various parameters in table 1, and the estimated values of parameters in Hsu Chen-Min and Hong Rong-Yan (2008) to serve as a reference of parameter settings, then using HP Filter Method to screen out seasonal fluctuations in quarterly data of per capita real output of Taiwan from 1982:Q1 to 2007:Q4. Adopting Bayesian estimation to derive estimates through 50,000 computer simulations, as
Table 1 Parameters Estimated by Dynare Built-in Bayesian Estimation Function

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Description</th>
<th>Prior Estimate</th>
<th>Posterior Estimate</th>
<th>CI</th>
<th>Prior Distribution</th>
<th>Standard Deviation</th>
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</thead>
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<td>$\alpha$</td>
<td>Degree of openness</td>
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<td>0.6255</td>
<td>0.3770</td>
<td>beta</td>
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<td>$\beta$</td>
<td>Discounted factor</td>
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<td>0.9606</td>
<td>beta</td>
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<td>$\sigma$</td>
<td>Elasticity of substitution of consumption (Reciprocal)</td>
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<td>0.6182</td>
<td>gamma</td>
<td>0.25</td>
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<td>$\eta$</td>
<td>Elasticity of substitution domestic and imported goods</td>
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<td>1.0771</td>
<td>0.6735</td>
<td>gamma</td>
<td>0.25</td>
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<tr>
<td>$\varphi$</td>
<td>Elasticity of substitution of labor supply (Reciprocal)</td>
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<td>0.9784</td>
<td>gamma</td>
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<td>$\theta_p$</td>
<td>Price stickiness coefficient (Calvo parameter)</td>
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<td>0.1486</td>
<td>beta</td>
<td>0.2</td>
</tr>
<tr>
<td>$\theta_i$</td>
<td>Imported goods price pass-through coefficient</td>
<td>0.5</td>
<td>0.4977</td>
<td>0.2030</td>
<td>beta</td>
<td>0.2</td>
</tr>
<tr>
<td>$\omega_d$</td>
<td>Domestic goods backward pricing proportion</td>
<td>0.5</td>
<td>0.4860</td>
<td>0.2775</td>
<td>beta</td>
<td>0.2</td>
</tr>
<tr>
<td>$\omega_f$</td>
<td>International goods backward pricing proportion</td>
<td>0.5</td>
<td>0.4899</td>
<td>0.1910</td>
<td>beta</td>
<td>0.2</td>
</tr>
<tr>
<td>$\rho_o$</td>
<td>Taylor Rule parameters of output gap</td>
<td>0.5</td>
<td>0.5572</td>
<td>0.2029</td>
<td>gamma</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>Taylor Rule parameters of price inflation</td>
<td>1.5</td>
<td>1.5389</td>
<td>1.2357</td>
<td>gamma</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>The public’s expectation of price inflation</td>
<td>1.5</td>
<td>1.4585</td>
<td>1.0733</td>
<td>gamma</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>The public’s expectation of output gap</td>
<td>0.5</td>
<td>0.4919</td>
<td>0.1428</td>
<td>gamma</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>Central bank policy smoothing parameter</td>
<td>0.5</td>
<td>0.6157</td>
<td>0.4079</td>
<td>beta</td>
<td>0.2</td>
</tr>
<tr>
<td>$\rho_T$</td>
<td>Central bank policy transparency parameter</td>
<td>0.5</td>
<td>0.5376</td>
<td>0.2113</td>
<td>beta</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Table 2 Characteristics of Adopted Shock Estimated by Dynare Built-in Bayesian Estimation Function

<table>
<thead>
<tr>
<th>Shock</th>
<th>Description</th>
<th>Prior Estimate</th>
<th>Posterior Estimate</th>
<th>CI</th>
<th>Prior Distribution</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_{te}$</td>
<td>Technological progress</td>
<td>0.01</td>
<td>0.0048</td>
<td>0.0025</td>
<td>Inv-gamma</td>
<td>Inf</td>
</tr>
<tr>
<td>$\varepsilon_{fo}$</td>
<td>Foreign output</td>
<td>0.01</td>
<td>0.0066</td>
<td>0.0026</td>
<td>Inv-gamma</td>
<td>Inf</td>
</tr>
<tr>
<td>$\varepsilon_t$</td>
<td>Terms of trade</td>
<td>0.01</td>
<td>0.0068</td>
<td>0.0026</td>
<td>Inv-gamma</td>
<td>Inf</td>
</tr>
<tr>
<td>$\varepsilon_{fr}$</td>
<td>Foreign interest rate</td>
<td>0.01</td>
<td>0.0045</td>
<td>0.0025</td>
<td>Inv-gamma</td>
<td>Inf</td>
</tr>
<tr>
<td>$\varepsilon_{pi}$</td>
<td>Foreign price inflation</td>
<td>0.01</td>
<td>0.0042</td>
<td>0.0024</td>
<td>Inv-gamma</td>
<td>Inf</td>
</tr>
<tr>
<td>$\varepsilon_{dc}$</td>
<td>Domestic cost</td>
<td>0.01</td>
<td>0.0057</td>
<td>0.0026</td>
<td>Inv-gamma</td>
<td>Inf</td>
</tr>
</tbody>
</table>

shown in table 1.
Figure 2  Prior Distribution (dashed line) and Posterior Distribution after Adding Reference Data (solid line)
3.6. Best Policy and Social Welfare Loss

In a small open economy, the central bank only has to conduct adjustment for domestic output and price (at this time foreign price is an exogenous variable in small economies), so the optimal policy is to carry out adjustment for fluctuations of domestic output and price. After forced $\pi_0$ equal to 0, through Philips curve domestic output will be in equilibrium state. This will make nominal side impact separate with real side, thus ensuring that the output gap is also equal to 0. This is the best policy, and social welfare losses are all deviation from this policy.

Gali and Monacelli (2005) and Woodford (2003) developed a social welfare loss function under the assumptions of their models, but it does not applicable to more generalized models. It is only applicable to model with most parameters near 1. According to Gali and Gertler (2007): “The model derived from Gali and Monacelli (2005) approximates the way of representative individual second-order social welfare loss, so can only be used under cases with special parameters, and can not accurately estimate more generalized models, including open economy model”.

In order to avoid that the divergent situation (which occurred when Dynare simulation is approaching end period) may affect the estimate of social welfare loss, we assume that the model has already converged after observing 50 periods. So we take the sum square of extents of deviation from equilibrium of price inflation and output gap of only first 50 periods as the

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measurements of social welfare loss, and assume that the influence of shocks after 50 periods is minimal, or can be ignored, in order to avoid the divergent situation at the end period of simulation (close to period 100) and the generated measurement error.

Therefore, this paper uses only the variations of deviation from equilibrium state, and their arithmetic sum to measure the social welfare loss. It should be noted that, when the evaluated social welfare losses are very close, we can not make an accurate conclusion that “policy with smaller social welfare loss is better”, and can only be used as a reference for analysis.

4. COMPARISONS OF MONETARY POLICY RULES

The estimated value of the central bank transparency measure parameter is 0.537. Simulating various shocks, we obtain the following results:

1) Technology shock

When an open economy faces positive shock of production side, it can make production technology upgrade, domestic price inflation decline, and there is a positive output gap. So the economy system responds with interest rate cut and currency appreciation. And the responses of various policies have relative degree of differences, among them the asymmetric policy rules D is the most efficient, as shown in table 3.

<table>
<thead>
<tr>
<th>Rule \ Variation</th>
<th>$\sum x_t^2$</th>
<th>$\sum \pi_t^2$</th>
<th>Loss</th>
<th>Order of Merits</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Inflation targeting</td>
<td>0.036</td>
<td>0.070</td>
<td>0.107</td>
<td>2</td>
</tr>
<tr>
<td>B Taylor rule</td>
<td>0.082</td>
<td>0.086</td>
<td>0.168</td>
<td>3</td>
</tr>
<tr>
<td>C Exchange-rate targeting</td>
<td>1.127</td>
<td>0.357</td>
<td>1.485</td>
<td>4</td>
</tr>
<tr>
<td>D when $x_t &lt; 0$, adopt a.</td>
<td>0.036</td>
<td>0.067</td>
<td>0.103</td>
<td>1</td>
</tr>
<tr>
<td>D when $x_t \geq 0$, adopt b.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2) Foreign income shock
Faced with a sudden increase of foreign income, the economy responds the exchange rate with a slight appreciation. But the excess demand of foreign economies for domestic goods still causes domestic price to rise, so the central bank raises interest rate. Under this situation, pegged exchange rate will not be able to avoid the fluctuations of foreign prices, but will only cause greater social welfare loss. As for rule D, it will also cause greater loss because of asymmetric preference negatively affecting economy in output gap. Overall, rule A has a smaller loss.

Table 4  Loss Comparison of Various Policy Rules
Facing Foreign Income Shock

<table>
<thead>
<tr>
<th>Rule</th>
<th>Variation</th>
<th>( \sum x_t^2 )</th>
<th>( \sum \frac{\pi_t^2}{\pi_t^*} )</th>
<th>Loss</th>
<th>Order of Merits</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Inflation targeting</td>
<td>0.022</td>
<td>0.043</td>
<td>0.066</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>Taylor rule</td>
<td>0.050</td>
<td>0.053</td>
<td>0.104</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>Exchange-rate targeting</td>
<td>0.699</td>
<td>0.221</td>
<td>0.921</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>when ( x_t &lt; 0 ), adopt a. when ( x_t \geq 0 ), adopt b.</td>
<td>0.052</td>
<td>0.057</td>
<td>0.109</td>
<td>3</td>
</tr>
</tbody>
</table>

3) Foreign terms of trade shock
When foreign terms of trade increases, that is, foreign terms of trade improves, this effect is similar to an increase of foreign income. It will cause foreigners to purchase more domestic goods, and result in appreciation of domestic currency, as well as inflation. The central bank will raise

Table 5  Loss Comparison of Various Policy Rules
Facing Terms of Trade Shock

<table>
<thead>
<tr>
<th>Rule</th>
<th>Variation</th>
<th>( \sum x_t^2 )</th>
<th>( \sum \frac{\pi_t^2}{\pi_t^*} )</th>
<th>Loss</th>
<th>Order of Merits</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Inflation targeting</td>
<td>0.033</td>
<td>0.064</td>
<td>0.097</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>Taylor rule</td>
<td>0.074</td>
<td>0.078</td>
<td>0.153</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>Exchange-rate targeting</td>
<td>1.030</td>
<td>0.326</td>
<td>1.357</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>when ( x_t &lt; 0 ), adopt a. when ( x_t \geq 0 ), adopt b.</td>
<td>0.077</td>
<td>0.084</td>
<td>0.161</td>
<td>3</td>
</tr>
</tbody>
</table>
interest rate. Faced with different policy rules, rule A is better. If we took C of exchange-rate targeting, it will cause significant fluctuations in output gap and domestic price.

4) Foreign interest rate shock

When foreign interest rate increases, that is, \( r < r^* \). This will result in devaluation of national currency through interest rate parity theory, and domestic goods become relatively inexpensive in the world markets, but cause domestic price inflation. Among various policy rules, the response of pegged exchange rate is best. It makes nominal impact not quickly spread to real economy. As for the policy generating largest loss, it is the asymmetric targeting rule.

| Table 6 Loss Comparison of Various Policy Rules Facing Foreign Interest Rate Shock |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Rule \ Variation | \( \sum x_i^2 \) | \( \sum \pi_{it}^2 \) | Loss | Order of Merits |
| A Inflation targeting | 3.881 | 5.212 | 9.093 | 2 |
| B Taylor rule | 8.437 | 6.439 | 14.877 | 3 |
| C Exchange-rate targeting | 0.008 | 0.079 | 0.087 | 1 |
| D when \( x_i < 0 \), adopt a. when \( x_i \geq 0 \), adopt b. | 8.849 | 7.072 | 15.921 | 4 |

5) Foreign inflation shock

When foreign price inflation rises, the reaction is that to avoid the impact,

| Table 7 Loss Comparison of Various Policy Rules Facing Foreign Inflation Shock |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Rule \ Variation | \( \sum x_i^2 \) | \( \sum \pi_{it}^2 \) | Loss | Order of Merits |
| A Inflation targeting | 2.025 | 3.850 | 5.876 | 2 |
| B Taylor rule | 4.459 | 4.629 | 9.088 | 4 |
| C Exchange-rate targeting | 1.816 | 4.482 | 6.298 | 3 |
| D when \( x_i < 0 \), adopt a. when \( x_i \geq 0 \), adopt b. | 2.017 | 3.764 | 5.782 | 1 |
the economy will respond with exchange rate appreciation to eliminate foreign price shock; at this time imported products are cheaper, but foreign demand for domestic output will decrease. This will reduce price inflation and thus cause interest rate cut to stimulate consumption and investment. Facing foreign inflation, the central bank also considers the impact on output at the same time, and thus will cause the economy more quickly converge to equilibrium.

6) Domestic cost shock

Facing an increase of domestic cost, this will cause the domestic Phillips curve to rise up. Therefore, domestic prices rise and the demand for domestic products decreases, while demand for foreign products increases, and exchange rate depreciates. Under this situation, the central bank could be easily forced to select price stabilization, and result in significant price inflation. From table 8 it can be found that in order to correct the output gap, it will pay a high cost in inflation (i.e., deviation of $\tilde{\pi}_t$ will be high). At this time the more effective policy is based on the policy of pegged exchange rate or pure price control.

<table>
<thead>
<tr>
<th>Rule \ Variation</th>
<th>$\sum x_t^2$</th>
<th>$\sum \tilde{\pi}_t^2$</th>
<th>Loss</th>
<th>Order of Merits</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Inflation targeting</td>
<td>64.353</td>
<td>59.596</td>
<td>123.950</td>
<td>3</td>
</tr>
<tr>
<td>B Taylor rule</td>
<td>84.183</td>
<td>1.418</td>
<td>85.602</td>
<td>2</td>
</tr>
<tr>
<td>C Exchange-rate targeting</td>
<td>82.326</td>
<td>2.354</td>
<td>84.681</td>
<td>1</td>
</tr>
<tr>
<td>D when $x_t &lt; 0$, adopt a. when $x_t \geq 0$, adopt b.</td>
<td>64.353</td>
<td>59.596</td>
<td>123.950</td>
<td>3</td>
</tr>
</tbody>
</table>

From the simulation results above, we had found that in a small open economy facing foreign interest rate and domestic cost shocks exchange-rate targeting is a good policy option. However, when it faces substantial impacts from terms of trade and foreign income shocks, exchange-rate targeting would inversely cause huge social welfare loss. After simulating
various shocks into various sectors, it was found that the policy having a better performance is inflation targeting. In particular, when the shocks are coming from abroad, including terms of trade, foreign income, foreign inflation, as well as technology. Hybrid or asymmetric inflation targeting is also crucial for technology and foreign shocks.

5. CONCLUSIONS AND RECOMMENDATIONS

This study suggests that central bank’s adoption of high transparency policy rules could avoid decision welfare loss. It is well-known that the time-consistency problem suggests that discretionary monetary policy can lead to poor economic outcomes. If monetary policy makers operate with discretion, they will be tempted to pursue overly expantionary monetary policies that boost employment in the short run but generate high inflation (and no higher employment) in the long run. A commitment to a policy rule like the Taylor rule or the inflation targeting rule solves the time-inconsistency problem. By following a set plan that does not allow policy makers to exercise discretion, they can achieve desirable long-run outcomes.

Even though there was no inflation during the last two decades, expantionary monetary policies to lower federal fund rate of policy instrument had created lots of bank excess reserves and therefore caused credit boom. It is an asset-price bubble, instead of currency bubble or hyperinflation, that came out.

It is due to Milton Friedman’s adage that in the long run, “Inflation is always and everywhere a monetary phenomenon”. Likewise, asset-price bubble is always and everywhere a bank credit phenomenon. As pointed out by John Taylor (2009, 2014), “the explanation for the financial crisis and weak recovery fits the facts of the past ten years very well. Deviations from good economic policy has been responsible for the very poor performance. Such policy deviations created a boom-bust cycle and were a significant in
the crisis and slow recovery”. In fact, Taylor further showed that such
deviations include the Fed’s low interest rate policy in 2003-2005 and lax
enforcement of financial regulations — both deviations from rule-based
policies that had worked in the past (Taylor, 2014, p. 63).

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