Open Economy Financial Instability *

Kenshiro Ninomiya **

A number of monetary crises have broken out in different parts of the world with one of them happening in Korea. Ito (1999) identified the fixed exchange rate system, capital flight, and financial fragility in domestic economies as underlying factors in these crises.

Asada (1995) studied both the fixed exchange rate and floating exchange rate systems by considering international capital mobility. Research showed that when international capital mobility is sufficiently high, the fixed exchange rate system destabilizes an economy whereas the floating exchange rate system brings about the opposite effect. Left out of this discussion was an examination of financial instability.

This paper will examine how international capital mobility and financial factors affect the dynamic fixed and floating exchange rate systems. This paper is to demonstrate how the stabilities of the systems depend on the level of risk faced by international lenders and the financial structure of the domestic economy. Results demonstrate that a stable financial structure is important to the Korean economy.

JEL Classification: F41, F31, F32
Keywords: financial instability, Minsky, Kaldorian dynamics, capital mobility

* Received March 29, 2007. Accepted October 27, 2007. The author wrote a part of this paper during his stay at the Commerce Division of Lincoln University as a visiting scholar. Thanks are due to Dr. Patrick Aldwell (Director of Commerce Division), Professor Paul Dalziel, and Professor Amal Sanyal for the warm support given. The author would also like to extend his gratitude to anonymous referees, Professor Takeshi Nakatani (Kobe University), Professor Peter Skott (University of Massachusetts, Amherst), Tadasu Matsuo (Kurume University) for the many valuable comments, and Ishii Memorial Securities Research Promotion Foundation for the financial support. Any remaining errors in this work are the responsibility of the author.

** Faculty of Economics, Shiga University, 1-1-1 Banba, Hikone, Shiga 522-8522, Japan, Tel: 81-749-27-1158, Fax: 81-749-27-1132, E-mail: k-nino@biwako.shiga-u.ac.jp
1. INTRODUCTION

A number of monetary crises have broken out recently in different parts of the world with Korea being one of the countries affected in 1997. When Minsky (1975) reevaluated the Keynesian Theory, the research introduced the financial instability hypothesis, proposing that the complicated financial structure underlying the capitalist economy generates business fluctuations.

The ideas of Minsky have been studied extensively. Taylor and O’Connell (1985) proved that an economy would fall into a financial crisis when a decline in expected profit rates aggravated the financial condition of firms and increased household preference for liquidity. Foley (1987), meanwhile, introduced the positive effects of liquid assets on investment into a model and showed that the model would give rise to a limit cycle.

The models by Taylor and O’Connell (1985) and Foley (1987) neglected to consider an open economy. Sethi (1992) extended the model by Foley (1987) into an open economy. The extended model found that an increase in the domestic money supply resulting from a current account surplus would destabilize an economy. Sethi (1992) offered suggestions that applied well to the financial crises that were to later strike in Asian countries such as Thailand, Indonesia, and Korea.

Yet Sethi (1992) considered only the case of the fixed exchange rate system without international capital mobility. The paper by Asada (1995) published several years later went on to consider international capital mobility and studied both the fixed exchange rate and floating exchange rate systems. Asada demonstrated that when international capital mobility is sufficiently high, the fixed exchange rate system destabilizes an economy whereas the floating exchange rate system imposes stability. Left out of the discussion was the issue of financial instability.

---

1) In recent works, for example, Asada (2006) and Ninomiya (2007) incorporated the dynamic equation of debt burden and discussed financial instability and cycles. They did not, however, consider an open economy.

2) Sarantis (1989, 1990-1991) considered the case of the floating exchange rate system with capital mobility.
Ito (1999) identified the (i) fixed exchange rate system, (ii) financial fragility in the domestic economy, and (iii) capital flight as factors underlying the Asian monetary crises. When the monetary crisis occurred in the Korean economy all three of these factors were present. Neither Sethi (1992) nor Asada (1995) considered these factors together in the earlier papers. Korea introduced the floating exchange rate system after experiencing the monetary crisis. Research now needs to consider financial instability in the floating exchange rate system.

This paper will begin by considering financial instability in the closed Kaldorian business-cycle model. Next, it will examine the degree to which international capital mobility affects the dynamic fixed and floating exchange rate systems as financial instability is considered.

The conclusion of this paper will reveal how the stabilities of the dynamic fixed and floating exchange rate systems depend on the degree of risk facing international lenders and the financial structure of the domestic economy. Results demonstrate that the stable financial structure is supremely important to the Korean economy.

2. THE BASIC MODEL OF FINANCIAL INSTABILITY

This study begins by examining financial instability in the closed Kaldorian business-cycle model. Many papers formulate a financial sector as the $LM$ equation. However, the $LM$ equation stabilizes an economy.\(^3\)

Rose (1969), Okishio (1986), and Ninomiya (2006) formulated the following equation to determine the interest rate $i$

$$EB = -(EX + EM) = -(I - S + M^d - M) = 0, \quad (1)$$

\(^3\) Chang and Smyth (1971) reformulated Kaldor (1940) by applying the Poincare-Bendixson theorem. Akashi and Asada (1985) introduced the $LM$ equation as a financial sector into the model reported by Chang and Smyth (1971). Asada (1995, 1997) proved the existence of the closed orbit by applying the Hopf-bifurcation theorem within the Kaldorian business-cycle model and then extended the model into an open economy.
where $EX$ is the excess demand for goods; $EB$, the excess demand for bonds; $EM$, the excess demand for money; $M$, the money supply; $M^d$, the demand for money; $S$, saving; and $I$, investment.

The main difference between the LM equation and equation (1) is that the latter contains saving, $S$, and investment, $I$. It is believed that the financial structure of the LM equation ignores several important aspects. $S$ and $I$ are factors of the goods market and are also deeply related to financial markets. Investment is the demand for loanable funds and saving is part of the supply of loanable funds. The interest rate must be affected by investment and saving unless the goods market is equilibrium $(I \neq S)$.

This paper proposes an oligopolistic economy and assume that the price level, $p$, is decided by the mark-up principle, as follows

$$p = \frac{(1 + \tau)WN}{Y},$$

(2)

where $\tau$ is the mark-up rate, $W$ is the nominal wage, $N$ is employment, and $Y$ is net income.

This paper assumes that there are rentiers and workers. The real wage income, $H_w$, is obtained from equation (2) as follows

$$H_w = \frac{WN}{p} = \frac{1}{1+\tau}Y.$$  

(3)

A percentage of the profits is assumed to be distributed to the rentiers. The share for the rentiers, $H_R$, is formulated as follows

---

4) In the IS-LM analysis, the dynamic equation for the interest rate is formulated as follows

$$i = F_i(EM) = F_i(M^{d} - M), F_i(0) = 0, F_i' > 0.$$  

(4.1)

Equation (4.1), however, depends on $EM + EB = 0$ and $EX = 0$. Okishio (1986) criticized equation (4.1) and formulated the dynamic equation of interest rate as

$$i = F_i(EB), F_i(0) = 0, F_i' < 0.$$  

(4.2)

Okishio (1986) described the macroeconomic model as the IS-BB analysis.
Open Economy Financial Instability

\[ H_R = \delta(Y - H_w) = \frac{\delta \tau}{1 + \tau} Y, \]  \hspace{1cm} (4)

where \( \delta \) is the ratio of the rentiers’ share.

The consumption function is assumed to be a linear function of \( H_w \) and \( H_R \). The consumption function and saving function are defined as follows

\[ C = c(H_w + H_k) + C_0 = c\left(1 + \frac{\delta \tau}{1 + \tau}\right)Y + C_0, \quad 0 < c < 1, C_0 > 0. \]  \hspace{1cm} (5)

\[ S = Y - C = \frac{(1-c) + (1-\delta c)\tau}{1 + \tau} Y - C_0 = sY - C_0, s > 0. \]  \hspace{1cm} (6)

The investment function, money demand function, and money supply function are defined as

\[ I = I(Y, K, i), I_Y > 0, I_K < 0, I_i < 0, \]  \hspace{1cm} (7)

\[ M^d = L(Y, i), L_Y > 0, L_i < 0, \]  \hspace{1cm} (8)

\[ M = \mu(Y, i)H, \mu_Y > 0, \mu_i > 0, \]  \hspace{1cm} (9)

where \( K \) is capital stock, \( \mu \) is a monetary multiplier, and \( H \) is high-powered money. Equation (7) is consistent with Kaldor’s discussion. \( L_Y < 0 \) indicates the following. Since the rise in income lessens the possibility of default, the demand for money may constitute a decreasing function of income. This effect expresses one of the “lenders’ risks.”

Money is a no-risk asset and a bond is a risky asset. The expected rate of return for holding a bond is defined as \( r_b = i - \nu f(Y) \) and the standard deviation is set as \( \sigma_b \). The parameter \( \nu \) denotes the degree of the risk of a default. The expected rate of return \( (r) \) and standard deviation \( (\sigma) \) for holding both money and bonds are
\( \mu_i > 0 \) means that the money supply increases as bank loans increase in an expanding economy. This is also one of the “lenders’ risks.”

By ordering equations (1), (6), (7), (8) and (9), we get

\[
EB = -[I(Y, K, i) - (sY - C_o) + L(Y, i) - \mu(Y, i)H] = 0. \tag{10}
\]

Solving equation (10) with respect to interest rate, \( i \), gives us

\[
i = i(Y, K, H), \tag{11}
\]

\[
r = (1 - \varepsilon)\sigma, \tag{5.1}
\]

\[
\sigma = (1 - \varepsilon)\sigma. \tag{5.2}
\]

Furthermore, we assume the following utility function

\[
U = r - \rho\sigma^2, \tag{5.3}
\]

where \( \rho \) expresses the degree of risk aversion. Substituting (5.1) and (5.2) into (5.3), we get the optimal portfolio \( \varepsilon^* \) as follows

\[
\varepsilon^* = \frac{vf(Y) - i}{2\rho\sigma^2} - 1 = \varepsilon(Y, i), \quad \varepsilon_y < 0, \quad \varepsilon_i < 0. \tag{5.4}
\]

This, in turn, leaves us with \( L_y < 0 \). This assumption is very similar to that of Rose (1969) and Taylor and O’Connell (1985).

\( \eta \) The excess reserves-deposit ratio \( \eta \) is assumed to be

\[
\eta = \eta(Y, i), \quad \eta_y < 0, \quad \eta_i < 0. \tag{6.1}
\]

For example, \( \eta_y < 0 \) means that commercial banks increase lending and reduce excess reserves when the economy is expanding. This is the same behavior described in footnote 5 and indicates the asset preference of banks. Based on equation (6.1), shows the following monetary multiplier

\[
M = \frac{1 + \theta}{\theta + \lambda + \eta(Y, i)} H = \mu(Y, i)H, \tag{6.2}
\]

where \( \theta(> 0) \) is the cash-deposit ratio and \( \lambda (0 < \lambda < 1) \) is the cash-reserve ratio.
Equation (11) shows that the interest rate, \( i \), may be a decreasing function of income, \( Y \). \( \phi \) turns out to depend on \( q \) and \( m_y \), and so on. If (for example) \( m_y < -q \), then \( \phi < 0 \) (see figure 1). \( \phi \) shows the financial structure in the domestic economy.

Considering the above discussion, there is a need to correct the Kaldorian business-cycle model as follows

\[
\dot{Y} = \alpha(C + I - Y), \quad \alpha > 0, \tag{12}
\]

\[
\dot{K} = I, \tag{13}
\]

\[
C = c \left( \frac{1 + \delta r}{1 + \tau} \right) Y + C_0, \quad 0 < c < 1, \tag{5}
\]

\[
I = I(Y, K, i), \quad I_y > 0, \quad I_K < 0, \quad I_i < 0, \tag{7}
\]

\[
i = i(Y, K, H), \quad i_y (= \phi) > 0, \quad i_K < 0, \quad i_H < 0. \tag{11}
\]

By ordering equations (5), (7), (11), (12) and (13), shows the following dynamic system \((S_a)\)

\[
\dot{Y} = \alpha \left[ c \left( \frac{1 + \delta r}{1 + \tau} \right) Y + C_0 + I(Y, K, i(Y, K, H)) \right] - Y, \quad (S_a.1)
\]

\[
\dot{K} = I(Y, K, i(Y, K, H)). \quad (S_a.2)
\]
The Jacobian matrix of the system (S_a) at the equilibrium point is expressed as

\[ J = \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix}, \]

(14)

\[ f_{11} = \alpha(q + I_i \phi), \quad f_{12} = \alpha(I_k + I_i) < 0, \]

\[ f_{21} = I_y + I_i \phi, \quad f_{22} = I_k + I_i < 0. \]

The characteristic equation of the dynamic system (S_a) is

\[ \lambda^2 + a_1 \lambda + a_2 = 0, \]

(15)
where,

\[-a_i = \text{trace}J_a = f_{11} + f_{22} = \alpha [q + I_i \phi] + (I_k + I_i k), \quad (16)\]

\[a_z = \det J_a = f_{11} f_{22} - f_{12} f_{21} = -\alpha s(I_k + I_i k) > 0. \quad (17)\]

From equation (16), the paper gets the following combination between \(m_y\) and \(q(=I_y - s)\), which satisfies \(a_i (= -\text{trace}J_a) = 0\)

\[m_y = \frac{L_q - \mu H}{I_i} q + \frac{(I_i + L_q - \mu H)(I_k + I_i k)}{\alpha I_i}. \quad (18)\]

Equation (18) can be drawn as figure 1. The paper defines \(m_{yi}\) as a variable which will satisfy \(a_i (= -\text{trace}J_a) = 0\) at any \(\alpha (= \alpha_a)\) or \(q (= q_a)\)

\[m_{yi} = \frac{L_q - \mu H}{I_i} q_a + \frac{(I_i + L_q - \mu H)(I_k + I_i k)}{\alpha_a I_i}. \quad (19)\]

The above discussion leads to the following Proposition 1.

**Proposition 1.** The dynamic system \((S_a)\) is locally stable if \(m_y > m_{yi}\). On the contrary, the dynamic system \((S_a)\) is locally unstable if \(m_y < m_{yi}\).

**Proof.** If \(m_y > m_{yi}\), which means that \(\alpha [q + I_i \phi] < -(I_k + I_i k)\), to obtain \(a_i > 0\). Likewise, exists \(a_z > 0\). The Routh-Hurwitz conditions for stable roots are satisfied. In contrast, if \(m_y < m_{yi}\), we get \(a_i < 0\) and the Routh-Hurwitz conditions for stable roots are not satisfied. Q.E.D.

The paper now examines Proposition 1 in an economic sense. \(q\) in figure 1 expresses a feature of the real sector, and \(m_y\) expresses a feature of the financial sector. Research finds from equation (11), on the other hand, that \(q\) is also deeply related to the financial sector. It is assumed that the
economy is at point $A$, that is, a point at which the dynamic system ($S_a$) is unstable. Since the original real factor makes the system stable ($q < 0$), the instability depends on only the financial factor.

The level of income, $Y$, will presumably diverge from the equilibrium point to an upper level as a result of an upset. If the risk of the lender declines with a drastic rise in income, the supply of loanable funds increases. As a result, the interest rate, $i$, will fall in spite of the rise in income. Since a fall in the interest rate promotes investment, $I$, the income, $Y$, will progressively increase. The dynamic system ($S_a$), on the other hand, is also unstable when the economy is at point $B$. The instability at point $B$, depends mainly on the real factors.\(^7\)

The interest rate will decline in spite of the rise in income in the area demarcated by the line ($m_y = -q$) to the left of the area containing point $A$. The financial structure is fragile when the economy is within this area. The financial structure of the Korean economy was fragile when the monetary crisis struck. With an increase in income, on the other hand, the interest rate will rise in the area demarcated by the line to the right of the area containing point $B$.\(^8\)

3. THE OPEN ECONOMY

Initially discussed was financial instability in the closed Kaldorian business-cycle model. The paper will now examine systems in an open economy. Research now moves to an open Kaldorian business-cycle model in which the financial instability is considered as follows

$$\dot{Y} = \alpha(C + I + J - Y), \quad \alpha > 0, \quad (20)$$

\(^7\) Asada(1995) examined only the diagonal line area in figure 1.

\(^8\) Ninomiya (2006) showed a limit cycle and demonstrated that the effect of a monetary policy depends on the financial structure.
\[ Q = \beta \left( i - \gamma g(Y) - r_f - \frac{\pi' - \pi}{\pi} \right), \quad \beta > 0, \quad \gamma > 0, \quad g'(=\kappa) < 0, \quad (21) \]

\[ A = J + Q, \quad (22) \]

\[ J = J(Y, \pi), \quad J_y < 0, \quad J_g > 0, \quad (23) \]

\[ C = c \left( \frac{1 + \delta r}{1 + r} \right) Y + C_0, \quad (5) \]

\[ I = I(Y, K, i), \quad I_y > 0, \quad I_K < 0, \quad I_i < 0, \quad (7) \]

\[ i = i(Y, K, H), \quad i_f (=\phi) > 0, \quad i_K < 0, \quad i_H < 0, \quad (11) \]

\[ \dot{K} = I, \quad (13) \]

where \( J \) is the balance of current account (net export); \( Q \), the balance of the capital account; \( A \), the total balance of payments; \( \pi \), the value of a unit of foreign currency in terms of domestic currency; \( \pi' \), the expected exchange rate in the near future; and \( r_f \), the expected rate of return for holding foreign bonds, excluding the influence of exchange risk. \(^9\) The parameter \( \beta \) represents the degree of international capital mobility, \( g(Y) \) represents the international lenders’ risk, and \( \gamma \) expresses the degree of the risk by international lenders. \(^10\)

Equation (21) reflects that the balance of the capital account determined by the difference between the domestic interest rate and expected rate of return of foreign bonds. Equation (22) is the definition of the total balance of payment.

---

\(^9\) \( r_f \) is assumed to be constant.

\(^10\) \( g(Y) \) and \( \gamma \) are almost the same as \( f(Y) \) and \( \nu \) in footnote 5.
3.1. The Case of the Fixed Exchange Rate System

Korea adopted the fixed exchange rate system during the monetary crisis. In the case of the fixed exchange rate system three equations are added

\[ \pi = \bar{\pi}, \quad (24) \]
\[ \pi^e = \pi, \quad (25) \]
\[ \dot{H} = A. \quad (26) \]

Equations (24) and (25) prove that the exchange rate is given. Equation (26) shows that the money supply becomes an endogenous variable under the fixed exchange rate system, unless the central bank adopts the so-called sterilization policy.

In the above equations (5), (7), (11), (13) and (20)-(26), the dynamic system of fixed exchange rates is complete. The dynamic system of fixed exchange rates \((S_b)\) are as follows

\[ \dot{Y} = \alpha \left[ c \left( \frac{1 + \delta r}{1 + r} \right) Y + C_0 + I(Y, K, i(Y, K, H)) + J(Y, \bar{\pi}) - Y \right] \quad (S_b.1) \]
\[ \equiv g_1(Y, K, H). \]

\[ \dot{K} = I(Y, K, i(Y, K, H)) \equiv g_2(Y, K, H). \quad (S_b.2) \]

\[ \dot{H} = J(Y, \bar{\pi}) + \beta [i(Y, K, H) - \gamma g(Y) - r] \]
\[ \equiv g_3(Y, K, H; \beta, \gamma). \quad (S_b.3) \]

The Jacobian matrix of the system \((S_b)\) at the equilibrium point can be expressed as
The characteristic equation of the dynamic system is

\[ \lambda^3 + b_1 \lambda^2 + b_2 \lambda + b_3 = 0, \]

where,

\[ b_1 = -g_{11} - g_{22} - g_{33} = -\alpha [q + J_y + I_i \phi] - (I_K + I_i K) - \beta i_t. \]  

\[ b_2 = g_{22}g_{33} - g_{23}g_{32} + g_{12}g_{33} - g_{13}g_{31} + (g_{11}g_{22} - g_{12}g_{21}) = [(\alpha [q + J_y] + I_i K) - \alpha I_i K \gamma \kappa] \beta - \alpha I_i K J_y + \alpha (-s + J_y)(I_K + I_i K). \]  

\[ b_3 = -(g_{12}g_{23} - g_{13}g_{22})g_{33} + (g_{11}g_{23} - g_{13}g_{21})g_{32} - (g_{12}g_{23} - g_{13}g_{21})g_{31} = -\alpha (-s + J_y) I_i K \beta. \]  

The paper defines \( m_{3b} \) as
In this case, \( m_{1b} \) satisfies \(-g_{11} - g_{22} = 0\) at any \( \alpha = \alpha_b \) or \( q = q_b \).

The above discussion proves the propositions below.

**Proposition 2.** The dynamic system of fixed exchange rates \( (S_y) \) is locally unstable when \( m_y < m_{1b} \) and the degree of international capital mobility is sufficiently low \( (\beta \to 0) \).

**Proof.** When \( m_y < m_{1b} \), (i.e.) \( \alpha[q + I_y] + J_y > -(I_k + I_i) > 0 \), shows \(-g_{11} - g_{22} < 0\). If the degree of international capital mobility is sufficiently low \( (\beta \to 0) \), then \( b_i < 0 \). In this case, the Routh-Hurwitz conditions are not satisfied. Q.E.D.

**Proposition 3.** The dynamic system of fixed exchange rates \( (S_y) \) is locally stable when \( m_y > m_{1b} \) and the degree of international capital mobility is sufficiently low \( (\beta \to 0) \).

**Proof.** (i) If \( m_y > m_{1b} \), then \(-g_{11} - g_{22} > 0\). If the degree of international capital mobility is sufficiently low \( (\beta \to 0) \), then \( b_i > 0 \) from equation (29).

(ii) When \( \beta \) is sufficiently small, then \( b_i > 0 \) from equation (30).

(iii) When \( \beta \) is sufficiently small, then \( g_{31} \to J_y \), \( g_{32} \to 0 \), and \( g_{33} \to 0 \). This, in turn, leaves

\[
b_1b_2 - b_3 = (g_{12}g_{21} + g_{13}g_{31} - g_{11}g_{22})(g_{11} + g_{22}),
\]

where,

\[
g_{13}g_{21} + g_{13}g_{31} - g_{11}g_{22} = \alpha I_iJ_y - \alpha(-s + J_y)(I_k + I_i) < 0.
\]

When \( m_y > m_{1b} \), then \( g_{11} + g_{22} < 0 \). Thus, \( b_1b_2 - b_3 > 0 \) if \( m_y > m_{1b} \).
The above discussion shows $b_1 > 0$, $b_2 > 0$, $b_3 > 0$, and $b_2 - b_1 > 0$ when $m_y > m_{1b}$ and the degree of international capital mobility is sufficiently low ($\beta \to 0$). The Routh-Hurwitz conditions are satisfied in this case. Q.E.D.

Explained in the previous section, $m_y$ is expressed as a feature of the financial sector. Propositions 2 and 3 demonstrate that the stability of the dynamic system $(S_b)$ depends on $m_y$ via the same mechanism as that described in the previous section when the degree of international capital mobility is sufficiently low ($\beta \to 0$).

Proposition 4 below is proved by considering the international capital mobility ($\beta > 0$).

**Proposition 4.** The degree of international capital mobility is assumed to be sufficiently high ($\beta \to \infty$). Even if $m_y < m_{1b}$, the dynamic system of fixed exchange rates $(S_b)$ becomes locally stable when the risk of international lenders is sufficiently small ($\gamma \to 0$) and $\alpha(q + J_\gamma) + I_K < 0$.

**Proof.** It is assumed that $\alpha(q + J_\gamma) + I_K < 0$ and the risk of international lenders is sufficiently small ($\gamma \to 0$).

(i) Then $b_1 > 0$ from equation (29) when the degree of international capital mobility is sufficiently high ($\beta \to \infty$).

(ii) Then $b_2 > 0$ from equation (30) when $\beta$ is sufficiently large.

(iii) Regarding $b_1 b_2 - b_3$, we obtain

$$b_1 b_2 - b_3 = \eta_1 \beta^2 + \eta_2 \beta + \eta_3,$$

where

$$\eta_1 = [-(\alpha[q + J_\gamma] + I_K) + \alpha(J_\gamma + I_K)^2].$$

Then $\eta_1 > 0$ when $\gamma$ is sufficiently small and $\alpha(q + J_\gamma) + I_K < 0$.  

Then \( h_3 b_2 - b_1 > 0 \) if \( \beta \) is sufficiently large.

The above discussion gives us \( h_1 > 0, \ b_2 > 0, \ b_3 > 0, \) and \( h_3 b_2 - b_1 > 0 \). The Routh-Hurwitz conditions are satisfied in this case. Q.E.D.

Under the condition \( \alpha(q + J_f) + I_k < 0 \), the instability of the dynamic system \((S_b)\) arises solely from the financial factor for the same reason described in the previous section. The domestic financial structure is fragile. Proposition 4 shows that the dynamic system \((S_b)\) becomes stable under the condition \( \alpha(q + J_f) + I_k < 0 \) when the degree of international capital mobility is sufficiently high.

This works via the following mechanism. The level of income \( Y \) will presumably diverge from the equilibrium point to an upper level as a result of an upset. If the risk of the domestic lenders declines with a drastic rise in income, the supply of loanable funds will increase. As a result, the domestic interest rate \( i \), will fall in spite of the rise in income. The domestic interest rate will presumably fall below the expected rate of return for holding foreign bonds \( r_f \). If the degree of international capital mobility is high enough (\( \beta \rightarrow \infty \)), the decline triggers a rapid capital outflow overseas, hence the domestic money supply is inclined to decrease. Consequently, the investment demand \( I \), will be restrained by the rise in the domestic interest rate and the domestic income will begin to decline. These are stabilizing effects on the economy. The dynamic system \((S_b)\) becomes stable in this case.

\[
Y \uparrow \Rightarrow i \downarrow \Rightarrow i < r_f \Rightarrow H \downarrow \Rightarrow i \uparrow \Rightarrow I \downarrow \Rightarrow Y \downarrow.
\]

The instability of the dynamic system \((S_b)\) is caused by the real factor when \( \alpha(q + J_f) + I_k > 0 \). In this case, the high international capital mobility cannot correct the instability in the system.

It is possible to use the Hopf-bifurcation theorem\(^{11)\) to prove Proposition 5

---

\(^{11)\) See Gandolfo (1997).
below when $\alpha(q + J_{y}) + I_{k} < 0$. As mentioned the domestic financial structure is fragile when $\alpha(q + J_{y}) + I_{k} < 0$. The following mathematical lemma is useful to apply as the proof for Proposition 5.

**Lemma.** 12) The characteristic equation $\lambda^{3} + b_{2}\lambda^{2} + b_{2}\lambda + b_{3} = 0$ has a pair of purely imaginary roots $\pm hi(i=\sqrt{-1}, \ h \neq 0)$ if and only if $b_{2} > 0$ and $b_{2}b_{2} - b_{3} = 0$ are satisfied. This case leaves the explicit solutions $\lambda = -b_{1}, \pm \sqrt{b_{2}i}$.

**Proposition 5.** The following are taken as assumptions in the dynamic system of fixed exchange rates ($S_{c}$): $m_{1} < m_{ib}$, $\alpha(q + J_{y}) + I_{k} < 0$, and the degree of risk by international lenders is sufficiently small ($\gamma \to 0$). There is one parameter value $\beta_{0}$ at which the Hopf-bifurcation occurs. When $\beta$ is close to $\beta_{0}$, there is at least one closed orbit around the equilibrium in the system ($S_{c}$).

**Proof.** See the Appendix.

The Korean economy showed a fragile financial structure and was destabilized by financial factors. Proposition 4 is not consistent with the monetary crisis actually experienced in Korea. On this basis it is possible to prove Proposition 6 below.

**Proposition 6.** The dynamic system of fixed exchange rates ($S_{c}$) is locally unstable when the degree of risk by international lenders is sufficiently large ($\gamma \to \infty$).

**Proof.** $\eta_{1} < 0$ when $\gamma$ is sufficiently large. If $\beta$ is sufficiently large, then $b_{2}b_{2} - b_{3} < 0$ and the Routh-Hurwitz conditions are not satisfied in this case. Q.E.D.

12) Asada (1995) gives the proof of lemma. $b_{2} > 0$ and $b_{2}b_{2} - b_{3} = 0$ imply one of the properties of Hopf-bifurcation from this lemma.
Proposition 6 holds that even if the degree of international capital mobility is sufficiently high \((\beta \to \infty)\) and \(\alpha(q + J_r) + I_k < 0\), the dynamic system \((S_0)\) is unstable. It is assumed that the level of income, \(Y\), will diverge from the equilibrium point to a lower level as a result of an upset akin to a monetary crisis, such as the crisis that struck in Thailand. A decline in domestic income is very likely to trigger a default on the domestic bonds, and the risk of international lenders accordingly rises \((\gamma \to \infty)\). These circumstances lead to capital outflows and a depressed economy. This mechanism is consistent with the experience of the monetary crisis in Korea.

Yet even with low international capital mobility \((\beta \to 0)\), the destabilizing influence of the domestic financial structure will prevent economic stability.

(i) \(\beta \to \infty\) \(Y \downarrow \Rightarrow \gamma \eta \uparrow (> i \uparrow) \Rightarrow i - \gamma \eta < r_f \Rightarrow H \downarrow \Rightarrow i \uparrow \Rightarrow I \downarrow \Rightarrow Y \downarrow\)

(ii) \(\beta \to 0\) \(Y \downarrow \Rightarrow i \uparrow \Rightarrow I \downarrow \Rightarrow Y \downarrow\)

3.2. The Case of the Floating Exchange Rate System

The paper will now examine the floating exchange rate system, the system Korea adopted after experiencing the monetary crisis. It is possible to formulate the model by adding three equations into equations (5), (7), (11), (13), and (20)-(23)

\[
A = 0, \quad (33)
\]

\[
\pi^* = \gamma(\pi - \pi^*) = 0, \quad (34)
\]

\[
H = \bar{H}. \quad (35)
\]

Equation (33) represents the equilibrium of the total balance of payment. Equation (34) formalizes the adaptive expectation hypothesis concerning the expected exchange rate. Equation (35) indicates that the money supply
becomes an exogenous variable in the floating exchange rate system.

The following dynamic system can be obtained by ordering equations (5), (7), (11), (13), (20)-(23), and (33)-(35)

\[
\dot{Y} = \alpha \left[ c \left( \frac{1 + \delta r}{1 + \tau} \right) Y + C_0 + I(Y, K, i(Y, K, H)) + J(Y, \pi) - Y \right], \quad (36)
\]

\[
\dot{K} = I(Y, K, i(Y, K, H)), \quad (37)
\]

\[
A = J(Y, \pi) + \beta \left[ i(Y, K, H) - \gamma g(Y) - r_f - \frac{\pi'}{\pi} + 1 \right], \quad (38)
\]

\[
\pi' = \rho(\pi - \pi') \quad \rho > 0. \quad (39)
\]

Solving equation (38) with respect to \( \pi \) gives us the following equation

\[
\pi = \pi(Y, K, \pi'), \quad (40)
\]

\[
\pi_Y = -\frac{J_\pi \pi + \beta(\phi + \gamma \kappa)\pi}{J_x \pi + \beta}, \quad \pi_K = -\frac{\beta \pi}{J_x \pi + \beta} > 0, \quad \pi'' = \frac{\beta}{J_x \pi + \beta} > 0.
\]

By substituting equation (40) into equations (36) and (39), shows the dynamic system for the floating exchange rates \( S_e \), as follows

\[
\dot{Y} = h_1(Y, K, \pi'; \beta, \gamma), \quad (S_e.1)
\]

\[
\dot{K} = I(Y, K, i(Y, K, H)) = h_2(Y, K), \quad (S_e.2)
\]

\[
\pi' = \rho[\pi(Y, K, \pi') - \pi'] = h_3(Y, K, \pi'; \beta, \gamma). \quad (S_e.3)
\]
The Jacobian matrix of this system is given by

\[
J_i = \begin{pmatrix}
  h_{i1} & h_{i2} & h_{i3} \\
  h_{i1} & h_{i2} & 0 \\
  h_{i1} & h_{i2} & h_{i3}
\end{pmatrix},
\]

(41)

\[h_{i1} = \alpha[q + \lambda_I + \lambda_J + \lambda_{\pi J}], \quad h_{i2} = \alpha(\lambda_K + \lambda_I + \lambda_{\pi K}),\]

\[h_{i3} = \alpha\lambda_{\pi J}, \quad h_{i21} = \lambda_I + \lambda_{\pi J}, \quad h_{i22} = \lambda_K + \lambda_I < 0,\]

\[h_{i31} = \rho\lambda_{\pi J}, \quad h_{i32} = \rho\lambda_{\pi K} > 0, \quad h_{i33} = \rho \left( -\lambda_{\pi J} \right) \left( \lambda_{\pi K} + \beta \right).
\]

The characteristic equation of this system is

\[\lambda^3 + e_1\lambda^2 + e_2\lambda + e_3 = 0,\]

(42)

where,

\[e_1 = -h_{i1} - h_{i2} - h_{i3},\]

(43)

\[e_2 = h_{i1}h_{i2} - h_{i2}h_{i3} + h_{i1}h_{i3} - h_{i1}h_{i2} + h_{i2}h_{i3},\]

(44)

\[e_3 = -h_{i1}h_{i2}h_{i3} + h_{i1}h_{i2}h_{i3} - h_{i1}h_{i2}h_{i3} + h_{i1}h_{i2}h_{i3}.
\]

(45)

To begin with, we briefly investigate the case where the international capital mobility is sufficiently low \((\beta \to 0)\). When \(\beta\) is small enough, then \(\pi_y \to -J_y J_z\), \(\pi_K \to 0\), and \(\pi_{\pi J} \to 0\). In this case, \(e_1 = -\alpha(q + \lambda_{\pi J}) - (\lambda_K + \lambda_I + \rho).\) If \(m_y < 0\) and \(|m_y|\) is large enough and proves that the dynamic system \((S_c)\) is unstable.

Next to examine is the case where the international capital mobility is
sufficiently high \((\beta \to \infty)\). In this case

\[
e_i = -\alpha \left[ \frac{1}{J_x \pi + \beta} \left[ -J_x \pi \kappa \gamma \beta + \cdots \right] \right] - (I_s + I_i K_i), \tag{46}
\]

\[
e_2 = \alpha \frac{\rho}{(J_x \pi + \beta) \gamma} [(-s + J_y + J_z \pi (\phi + \gamma \kappa) + (I_I + I_i K_i \pi + J_z \rho \pi (\phi + \gamma \kappa)) \beta^2 + \cdots], \tag{47}
\]

\[
e_3 = \frac{\rho}{J_x \pi + \beta} \left[ -(I_K + I_i K_i)(\phi + \gamma \kappa) \pi + (I_I + I_i K_i \pi + \beta) \right], \tag{48}
\]

\[
e_1 e_2 - e_3 = \frac{\alpha^2 \rho J_x^2}{(J_x \pi + \beta)^3} [(\phi + \gamma \kappa)^2 \pi^2 \beta^3 + \cdots]. \tag{49}
\]

The following proposition can be proved from equation (47).

**Proposition 7.** The international capital mobility is assumed to be sufficiently high \((\beta \to \infty)\) in the dynamic system of floating exchange rates \((S_c)\). In this case, the system \((S_c)\) is unstable when \(\phi < 0 (m_r < -q)\) and the degree of risk by international lenders is sufficiently small \((\gamma \to 0)\).

**Proof.** The paper proposes that the degree of international capital mobility is sufficiently high \((\beta \to \infty)\). If the risk by international lenders is sufficiently small \((\gamma \to 0)\) and \(\phi < 0\), then there is a negative coefficient of \(\beta^2\) from equation (47). The Routh-Hurwitz conditions are not satisfied in this case, since \(e_2 < 0\). Q.E.D.

The paper will now examine Proposition 7. The level of domestic income \(Y\) is assumed to diverge from the equilibrium point to a lower level as a result of an upset. The decrease in domestic income \(Y\) improves the balance of a current account because of the decrease in imports. In this case
the exchange rate $\pi$ decreases, inducing a further decrease in income.

On the other hand, the fall in domestic income $Y$, will enhance the domestic interest rate, $i$, when $\phi < 0$. This means that the domestic interest rate $i$ will tend to rise over the expected rate of return for holding foreign bonds $r_f$. If the international capital mobility is high enough, the exchange rate $\pi$ will decrease with the capital inflow from overseas. A fall in net exports will decrease the domestic income $Y$. This shows that the dynamic system of the floating exchange rates is not always stable when the international capital mobility is sufficiently high.

$$Y \downarrow \Rightarrow i \uparrow \Rightarrow i > r_f \Rightarrow Q \uparrow \Rightarrow \pi \downarrow \Rightarrow J \downarrow \Rightarrow Y \downarrow$$

Proposition 8 can be proved when the risk by international lenders is sufficiently large ($\gamma \to \infty$) or $\phi > 0$.

**Proposition 8.** The international capital mobility is assumed to be sufficiently high ($\beta \to \infty$) in the dynamic system of floating exchange rates $(S_c)$. In this case, the system $(S_c)$ is stable when the risk by international lenders is sufficiently large ($\gamma \to \infty$).

**Proof.** The degree of international capital mobility is assumed to be sufficiently high ($\beta \to \infty$). If the risk by international lenders is sufficiently large ($\gamma \to \infty$), then equations (46) to (49) gives $e_1 > 0$, $e_2 > 0$, $e_3 > 0$, and $e_1e_2 - e_3 > 0$. Thus, the Routh-Hurwitz conditions are satisfied in this case. Q.E.D.

Proposition 8 shows that the dynamic system of floating exchange rates $(S_c)$ is stable under a condition of high international capital mobility even when the financial structure is fragile. The level of domestic income $Y$ will presumably diverge from the equilibrium point to a lower level as a result of an upset. The possibility of a default on domestic bonds increases at such times. If the domestic interest rate $i - \gamma \eta$ tends to fall below the expected
rate of return for holding foreign bonds \( r_f \), then the exchange rate \( \pi \) will rise with the capital outflow. The domestic income \( Y \) will progressively increase as the net exports increase.

\[
Y \downarrow \Rightarrow (\gamma i \uparrow) \Rightarrow i - \gamma \eta < r_f \Rightarrow Q \downarrow \Rightarrow \pi \uparrow \Rightarrow J \uparrow \Rightarrow Y \uparrow
\]

Important to note is that the stability of this system depends on the degree of risk by international lenders. The risk by international lenders is generally high in an economy with a fragile financial structure. This means that economic stability conceals the fragility of a domestic economy. If the risk by international lenders falls to a low level in such a case, the economy will become unstable. A stable financial structure is supremely important to the Korean economy in every sense.

4. CONCLUSION

Numerous monetary crises have broken out globally and Korea has experienced one of them. Ito (1999) identified the fixed exchange rate system, capital flight, and financial fragility in domestic economies as factors underlying these crises.

Asada (1995) introduced imperfect international capital mobility into the Kaldorian business-cycle model in an open economy and examined how capital mobility affects the dynamic systems of fixed and floating exchange rates. This research showed that when the degree of international capital mobility is sufficiently high, the fixed exchange rate system destabilizes an economy whereas the floating exchange rate system imposes economic stability but did not discuss the financial instability proposed by Minsky.

This paper examined financial instability within the closed and open Kaldorian business cycle models and reconsidered how the degree of international capital mobility affects the dynamic systems of fixed and floating exchange rates. The main conclusions of this paper are as follows:
(i) The instability of the closed and open dynamic systems is triggered only by financial factors \( m_{x} \).

(ii) There exists at least one closed orbit around the equilibrium in the dynamic system of fixed exchange rates \( (S_{b}) \) under certain conditions.

(iii) The stabilities of dynamic systems of fixed and floating exchange rates depend on the degree of risk by international lenders and the financial structure of the domestic economy.

These conclusions depend mainly on the considerations of financial instability in the dynamic system and show that a stable financial structure is important to the Korean economy. This conclusion leads to one of the main agendas for further research as it is noted that debt burden is not incorporated within the model that has led to this conclusion. New research must examine the financial instability hypothesis and formal mathematical models on related topics in treating the cumulative debt burden as one of the reasons for financial instability.

**APPENDIX**

**Proof of Proposition 5\(^{13}\)**

It is suppose \( m_{x} < m_{y} \), \( \alpha[I_{y} + I_{y}I_{x}] + I_{k} < 0 \) and the degree of risk by international lenders is sufficiently small \( (\gamma \rightarrow 0) \).

If \( \beta \) is sufficiently large, we have \( h_{b_{x}}b_{2} - b_{3} > 0 \) from the proof of Proposition 4. On the contrary, if \( \beta \) is sufficiently small, then \( h_{b_{x}}b_{2} - b_{3} < 0 \) from the proof of Proposition 3. Since \( h_{b_{x}}b_{2} - b_{3} \) is the smooth and continuous function with \( \beta \), there exists at least one value \( \beta_{0} \) at which \( h_{b_{x}}b_{2} - b_{3} = 0 \) and \( \partial(h_{b_{x}}b_{2} - b_{3})/\partial\beta|_{\beta=\beta_{0}} \neq 0 \). Then \( b_{2} > 0 \).

It follows from the lemma that the characteristic equation has a pair of imaginary roots \( \lambda_{1} = \sqrt{b_{2}}i \), \( \lambda_{2} = -\sqrt{b_{2}}i \) at \( \beta = \beta_{0} \). From the Orlando

\(^{13}\)The method of the proof is based on Asada (1995) and Yoshida (1999).
formation then
\[ b_1 b_2 - b_3 = -(\lambda_1 + \lambda_2)(\lambda_2 + \lambda_3)(\lambda_3 + \lambda_4) = -2h_1(\lambda_2^3 + 2h_1 \lambda_2 + h_1^2 + h_2^2) \]

Differentiating this equation with \( \beta \) then

\[ \frac{\partial (b_1 b_2 - b_3)}{\partial \beta} = -2 \left[ \frac{\partial h_1}{\partial \beta} \left( \lambda_2^2 + 2h_1 \lambda_2 + h_1^2 + h_2^2 \right) \right]. \]

Substituting \( h_1 = 0 \) and \( h_2 = h \) into the above equation then

\[ \frac{\partial (b_1 b_2 - b_3)}{\partial \beta} \bigg|_{\beta = \beta_0} = -2(\lambda_2^2 + h^2) \left[ \frac{\partial h_1}{\partial \beta} \bigg|_{\beta = \beta_0} \right], \]

where, \( h_1 \) is the real part of two complex conjugate numbers and \( h_2 \) is the absolute value of the imaginary part. Therefore, if

\[ \frac{\partial (b_1 b_2 - b_3)}{\partial \beta} \bigg|_{\beta = \beta_0} \neq 0 \text{ then } \frac{\partial h_1}{\partial \beta} \bigg|_{\beta = \beta_0} \neq 0. \]

From the above discussion, all of the conditions in which Hopf-bifurcation occurs are satisfied at the point \( \beta = \beta_0 \). Q.E.D.

REFERENCES

__________, Macrodynamics of Growth and Cycles, Nihon Keizai


Rose, Hugh, “Real and Monetary Factors in the Business Cycle,” Journal of


