Non-Periodic Cycles and Long Memory Property in the Korean Stock Market

Seong-Min Yoon** · Sang Hoon Kang***

This paper presents evidence on long memory and the identification of non-periodic cycles in Korean stock market returns. Adjusted $R/S$ analysis is used to distinguish how the stock returns differ from an independent and identically distributed series. Unlike standard techniques that identify only regular periodic cycles, this analysis is able to capture not only the long memory properties but also the presence of non-periodic cycles in the Korean stock market returns. The identification of non-periodic cycles (approximately two and four years in average duration) corresponds to the term of the business cycle of the Korean economy.

JEL Classification: C32, C52, G14

Keywords: long memory, rescaled range analysis, non-periodic cycle, KOSPI
1. INTRODUCTION

The efficient market hypothesis (EMH) postulates that prices always fully reflect all relevant information (Fama, 1970). The change in price is caused only by unexpected new information. The traditional definition of market efficiency assumes that the market is composed of homogenous traders who react to new information in a linear fashion. This implies that when new information arrives in the market, all investors react to information immediately, not in a non-linear fashion which accumulates to a time series of events. This linear paradigm assumes that asset returns are an independent and identically distributed (i.i.d.) series.

The linear paradigm has been widely criticized since empirical time series are found to be in favor of deterministic nonlinear behavior such as long memory which is one of typical forms of non-linear dynamics where irregular arrival of new information affects a time series in terms of correlating past returns with the returns of today. Long memory series also exhibit non-periodic cycles that persistence or patterns tend to reverse themselves less often than a random walk series. These non-periodic cycles correspond to market business cycles (Peters, 1991, 1994).

However, the one year duration of the non-periodic cycle is questionable for identifying an economic trend or cycle in the Korean stock market. The focus of this paper is to apply Peters’ classical adjusted R/S analysis to stock returns in an attempt to examine evidence of long memory and some type of non-periodic cycles in the KOSPI returns. In this respect, this paper provides two important contributions. First, it is apparent that stock returns are found to be not identically independent or distributed with a normal distribution, but exhibit a leptokurtic distribution which has fatter tails than the normal distribution. This result of this paper confirms the non-normality and non-linear dynamics of stock returns that is characterized by long memory properties.

This paper also examines the presence of long memory and finds possible market business cycles in the market. The test for long memory in the stock market index returns provides an important guideline of market efficiency since the validity of the efficient market hypothesis depends on the presence or absence of long memory in the stock returns. Additionally, for technical analysis, the identification of market cycles gives a potential opportunity to earn abnormal returns in the stock markets. The identification of market cycles may give rise to an opportunity to gain abnormal returns for technical analysis.

The paper is structured as follows. Section 2 discusses the model framework of rescaled range analysis. Section 3 provides descriptive statistics for two indices. In particular, the distribution of stock returns has an excess kurtosis and skewness that indicates that they appear to be far away from the assumption of normality. Section 4 presents the evidence of long memory property and non-periodic cycles in the stock markets. Section 5 contains the conclusions.
2. MODEL FRAMEWORK

2.1. Rescaled Range Analysis

Following Peters (1994), rescaled range analysis begins with a time series in prices of length $M$. The time series can be changed to a time series logarithmic returns of length $y = M - 1$

$$y_t = \log \left( \frac{M_{t+1}}{M_t} \right), \quad t = 1, 2, \ldots, (M - 1).$$ (1)

Divide this time period into $A$ contiguous sub-periods of length $n$, such that $A \cdot n = N$. Each sub-period is labeled $I_a$, with $a = 1, 2, \ldots, A$ and then each element in $I_a$ is labeled $y_{k,a}$, with $k = 1, 2, \ldots, n$. For each sub-period $I_a$ of length $n$ the average is calculated as

$$e_a = \frac{1}{n} \sum_{k=1}^{n} y_{k,a},$$ (2)

where $e_a$ is the average value of the $y_i$ contained in sub-period $I_a$ of length $n$. Then the study calculates the time series of accumulated departures $X_{k,a}$ from the mean at a given time $k$ for each sub-period $I_a$

$$X_{k,a} = \sum_{k=1}^{n} (y_{k,a} - e_a), \quad k = 1, 2, \ldots, n.$$ (3)

The range that the time series covers from maximum values to minimum values within $I_a$ is defined as

$$R_{I_a} = \text{Max}(X_{k,a}) - \text{Min}(X_{k,a}),$$ (4)

1) The use of contiguous sub-periods means there is no overlap in the data, i.e., $1, \ldots, n, n+1, \ldots, 2n, 2n+1, \ldots$
where $1 < k < n$, and the standard deviation for each sub-period $I_a$ can be expressed as

$$S_{I_a} = \left[ \frac{1}{n} \sum_{t=1}^{n} (y_{t,a} - e_a)^2 \right]^{1/2}. \quad (5)$$

The value of the range depends on each given time. The range for each sub-period ($R_{i_a}$) is then rescaled by the corresponding standard deviation ($S_{I_a}$) depending on $A$ contiguous sub-periods of length $n$. The average value of $R/S$ for sub-period length $n$ is estimated by

$$\frac{(R/S)_a}{A} = \frac{1}{A} \sum_{i=1}^{A} \left( \frac{R_{i_a}}{S_{I_a}} \right). \quad (6)$$

and this process from equation (1) to equation (6) must be repeated by increasing $n$ over successive values so that $(M - 1)/n$ is an integer value until $(M - 1)/2$. The Hurst exponent coefficient may be estimated using the OLS regression of the following form:

$$\log(R/S)_a = \log(c) + H \cdot \log(n) + \epsilon. \quad (7)$$

The log($R/S)_a$ is regressed, with log($n$) as the independent variable. The Hurst exponent is estimated as the slope of the equation. According to theory, the values of the Hurst exponent ($H$) are within the range $0 < H < 1$. There are three interpretations of the Hurst exponent ($H$): (i) $H = 0.5$ means time series are independent, random Gaussian process, but ‘it would also include non-Gaussian processes, such as the Student-t, or gamma’ (Peters, 1994, p. 61). (ii) If a value of the Hurst exponent ($H$) is in the range $0.5 < H < 1$, the time series exhibit persistence which is characterized by ‘long memory effect’. The persistence means that if the time series have been up or down, they are likely to continue to be up or
down in the future. (iii) Conversely, if $0 < H < 0.5$, the time series have anti-persistence. This implies that whenever the time series has been up in the last period, they are more likely to down in the future (Skjeltrop, 2000).

2.2. Power of $R/S$ Analysis

Early applications of $R/S$ analysis have a weakness with the lack of a significance test for the Hurst exponent. More advanced statistical techniques have been developed to evaluate the significance of $R/S$ analysis results. Anis and Lloyd (1976) developed an equation for calculating the expected $R/S$ values, $E(R/S)$ which is implied by a pure random process. This is expressed as

$$E(R/S)_e = \frac{\Gamma(0.5(n-1))}{\pi^{0.5}\Gamma(0.5n)} \sum_{r=1}^{n-1} \left[ (n - \gamma) / r \right]^{0.5}.$$ (8)

Peters (1994) argued that this equation had a missing error for small samples and a bias $(n < 20)$ using the Monte Carlo simulation. Peters proposed the modified equation with an error correction. The modified equation is derived as

$$E(R/S)_e = \left( \frac{n - 0.5}{n} \right) \sqrt{\frac{2}{n\pi}} \sum_{r=1}^{n} \sqrt{\frac{n-r}{n}}.$$ (9)

The Peter’s modified version is considered in this paper.2) Expected values of the Hurst exponent, $E(H)$, are generated by taking the ordinary least-square regression of $E(R/S)$ for the same sample range of $n$ as for the empirical $R/S$ values. Because the $R/S$ values are normally distributed with

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2) Couillard and Davison (2005) argued that the original Anis and Lloyd’s $E(R/S)$ model more accurate than Peters’ (1994) model to describe the asymptotic behaviour of the classical $R/S$ analysis. However, practically, both results are not significantly different (McKenzie, 2001).
Gaussian random variables, the values of $H$ are allowed to be normally distributed with expected variance $1/T$, where $T$ is the total number of observations in the sample.\textsuperscript{3)} The significance calculated by the standard deviations of a Gaussian random walk is expressed as

$$\sigma_H = \frac{H - E(H)}{\sqrt{1/T}}. \quad (10)$$

As mentioned above, $R/S$ analysis can also uncover non-periodic cycles in a time series. Even though long memory processes theoretically last forever, this memory might be finite. When long memory perishes, a time series begins to follow a random walk; that is, the trend will be changed, so called the crossover point. Technically, the estimation of the Hurst exponent uses the proper range for the linear relationship between $\log(R/S)_n$ versus $\log(n)$ values from equation (7). If there is a crossover point, this can be seen as a break point in the $\log(R/S)_n$ versus $\log(n)$ plot where the slope changes for a certain value, $\log(n_{\text{max}})$, indicating a non-periodic cycle with length equal to $n_{\text{max}}$. Practically, it is very difficult to identify the break point in the plot because the slope is close to that of a random walk. Peters (1994) proposed the $V$-statistic that is defined as

$$V_n = \frac{(R/S)_n}{\sqrt{n}}. \quad (11)$$

The ratio of $V$-statistic is such that the rescaled range is scaled by the square root of time. If the process is an independent and random process, the ratio will be constant and the slope of $V$-statistic plot is flat. If the process is persistent and the $R/S$ values are scaling at a faster rate than the square root time, and the slope of $V$-statistic plot is positively increasing. If the process is anti-persistent, the slope of $V$-statistic is negatively decreasing.

\textsuperscript{3)} This is tested by Peters (1994, p. 72) using the Monte Carlo simulation.
and long memory fades and the cycle ends. Subsequently, if the slope of the $V$-statistic plot is positively increasing again, another cycles exists (Peters, 1994). 4)

3. DATA

3.1. Data and Descriptive Statistics

This study uses a data set consisting of closing daily prices of the KOSPI as transacted on the KRX. 5) The KOSPI is a market value weighted index for all common stocks listed on the KRX, with a base value of 100 on January 4, 1980. The daily data cover the period from January 4, 1980 to January 31, 2005, equating to a total of 7,062 observations. Plots of the daily closing prices for the data set are shown in figure 1.

From the middle of the 1980s, the KOSPI value increased rapidly along with the economic growth in Korea. As a result of the October 1997 financial crisis in Korea, the KOSPI value followed a dramatic downward trend, until the middle of 1998; the index value then grew gradually following market reformation under IMF supervision. However, following the financial crisis, it is apparent that the cycle of KOSPI dynamics has becomes shorter and more volatile, in association with uncertainty over the rate of economic growth.

4) The $V$-statistic is capable of discerning cycles in the data. The cycles include not only periodic cycles but also non-periodic cycles. Deterministic non-linear dynamic systems may generate non-periodic cycles which have an average cycle length, but an erratic length for future cycle. Furthermore, the $V$-statistic plot can capture multiple cycles if they exist in the data. However, more than four cycles become indistinct indicating that the log ($R/S$) plot would be indistinguishable from the trend which is considered as indicative of no average cycle length (McKenzie, 2001).

5) Some empirical studies have suggested that monthly or weekly frequencies over longer period are more appropriate for the identification of non-periodic cycles in the market (McKenzie, 2001; Peters, 1994). Despite the lack of data history in the KRX, adjusted $R/S$ analysis is applied to only daily data in this paper.
Figure 1  Dynamics of Daily KOSPI

Note: The daily sample period covers from January 4, 1980 to January 31, 2005.
Source: KisValue (2006).

Table 1  Summary Statistics for Daily KOSPI Returns

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Skew.</th>
<th>Kurt.</th>
<th>J-B</th>
<th>Q(5)</th>
<th>Q(10)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.032</td>
<td>1.5</td>
<td>−0.15</td>
<td>7.68**</td>
<td>6483.06**</td>
<td>94.23**</td>
<td>101.48**</td>
</tr>
</tbody>
</table>

Note: This table describes several summary statistics, including mean (%), standard deviation (Std. Dev. %), skewness (Skew.), kurtosis (Kurt.), Jarque-Bera (J-B) test and Box-Pierce test Q(10) for the returns. ** indicates significance at the 5% level.

Table 1 provides descriptive statistics for the sample returns. The sample mean of returns is 0.032%, and the corresponding standard deviation of returns is 1.56%. Of interest is the fact that the stock returns appear to be extremely non-normal, displaying significant leptokurtosis. Skewness of the returns is insignificantly different from zero, while kurtosis is large and statistically significant, at the 5% level. 6) The Jarque-Bera (J-B) statistic for

6) Generally, a normal distribution is not skewed (skewness=0) and is defined to have a coefficient of kurtosis of 3.


Table 2  Stationarity Tests for Daily KOSPI Returns

<table>
<thead>
<tr>
<th>ADF</th>
<th>Without Trend</th>
<th>With Trend</th>
<th>PP</th>
<th>Without Trend</th>
<th>With Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>–59.20(1)***</td>
<td>–59.21(1)***</td>
<td>–75.45(13)***</td>
<td>–75.45(13)***</td>
<td></td>
</tr>
</tbody>
</table>

Note: Values in parentheses represent the number of lags used in the calculation of the ADF and PP test statistics. The ADF and PP critical values without trend: –3.44 at the 1% significance level; the ADF and PP critical values with trend: –3.96 at the 1% significance levels. All of the above estimated t-test statistics are smaller than the 1% critical value. *** indicate the rejection of t-statistic at the 1% significance level.

normality confirms that the returns depart significantly from those associated with a normal distribution.

Additionally, a common finding in time series regression is that residuals are correlated with dependent lagged values. This autocorrelation violates the assumption in time series models that returns are \( i.i.d. \) series. In order to test the null hypothesis of independence, Box-Pierce tests for serial correlation can be performed on the returns. Under the null hypothesis of no serial correlation, the test statistic is distributed asymptotically as a \( \chi^2 \) (Chi-square) distribution with 5 and 10 degrees of freedom, respectively. The results support the rejection of the null hypothesis; that is, significant dependence is present in the stock returns.

As argued by Klemes (1994), nonstationarity in the mean adds an upward bias to the estimated Hurst exponent. Accordingly, Augmented Dickey-Fuller (1979) and Phillips-Peron (1988) tests are used in this section to test the stationarity of sample stock returns. These tests can be estimated with or without an intercept and a trend term as well as for various lag lengths.

The null hypothesis of the ADF and PP tests is that a time series contains a unit root, an \( I(1) \) process, against the alternative of stationarity. The values of the t-test statistic reported in table 2 are smaller than the critical values, at the 1% significance level, allowing rejection of the null hypothesis of a unit root in stock returns. The sample returns are stationary.
Most standard statistical models for a financial time series are based on the random walk hypothesis, assuming that return series is a normal \( i.i.d. \) variable. However, the distributional nature of return series has been analyzed in many studies. For example, Mandelbrot (1963) has observed a fat-tailed distribution in cotton price returns. Meanwhile Fama (1965) has redefined the properties of stock return distributions, showing that these normally display negative skewness and higher kurtosis than a normal distribution.

One of the most common explanations of fat-tailed distributions is that new information arrives infrequently in stock markets (Skjeltrop, 2000). Theoretically, when information arrives, investors immediately react to it. In reality, investors have heterogeneous trading horizons that result in fat-tailed returns. For example, when new information arrives at financial markets,

**Figure 2  Distribution of Daily KOSPI Returns**

Note: The solid line is represented in terms of the Lorentz distribution. The returns are consistent with the Lorentz distribution rather than a normal one.
some investors react to it immediately, while others delay reactions until the new information has been confirmed, waiting until a trend emerges in the market.

Figure 2 displays the empirical distribution of KOSPI returns using the Gaussian distribution (dotted line) and the Lorentz distribution (solid line), which is characterized by a fat-tailed distribution.\(^7\) The distribution of the returns clearly reveals a fat-tailed distribution, since the frequency of returns (round spots) tracks the Lorentz distribution.

The KOSPI returns follow a non-normal distribution, making the assumption of a normal distribution inappropriate in modeling the distributional properties of KOSPI returns.\(^8\) Such a fat-tailed distribution is normally attributed to long memory in stock returns (Skjeltorp, 2000).

\section*{4. EMPIRICAL RESULTS}

Before doing \textit{R/S} analysis, it is an important rule of thumb to eliminate or at least minimize linear dependence since it can bias the estimate of the Hurst exponent and to classify a process as having a long memory when it is, in reality, a short memory process (Skjeltorp, 2000; Kang, 2006). Generally, financial time series of daily frequency exhibit significant autoregressive process which biases the result of the Hurst exponent. In order to overcome this problem, Peters (1994) suggested the first of autoregressive process AR(1) residuals to reduce the bias when doing \textit{R/S} analysis.\(^9\) Although

\(^7\) \(P(r) = f_r + \frac{2b}{\pi r^2 + a^2}\) where \(f_r = 0, a = 0.00111\) and \(b = 0.0016\) for the KOSPI returns.

\(^8\) Empirical studies show non-normal characteristics of financial returns using the popular Lévy stable distribution (Peters, 1994; Skjeltorp, 2000). However, Kim and Yoon (2004) simulate that the Lorentz distribution well fits the returns of yen-dollar exchange rate. Therefore, this study considers the Lorentz distribution for the KOSPI returns.

\(^9\) The AR(1) process used in the section follows the following equation

\[ y_t = 0.0003 + 0.1026 y_{t-1} + \epsilon_t \]

where \(y_t\) is the returns of KOSPI and \(\epsilon_t\) is a white noise error term. The estimated results are (the values of \(t\)-test statistics are in parentheses).
Table 3 Results of the Rescaled Range Analysis for Daily KOSPI Returns

<table>
<thead>
<tr>
<th>Sub-period Length (n)</th>
<th>Log (n)</th>
<th>Log R/S</th>
<th>Log E(R/S)</th>
<th>V-statistic R/S</th>
<th>V-statistic E(R/S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.0000</td>
<td>0.4508</td>
<td>0.4233</td>
<td>0.8930</td>
<td>0.8381</td>
</tr>
<tr>
<td>14</td>
<td>1.1461</td>
<td>0.5570</td>
<td>0.5305</td>
<td>0.9637</td>
<td>0.9067</td>
</tr>
<tr>
<td>20</td>
<td>1.3010</td>
<td>0.6583</td>
<td>0.6360</td>
<td>1.0180</td>
<td>0.9670</td>
</tr>
<tr>
<td>25</td>
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</tr>
<tr>
<td>28</td>
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<td>1.0141</td>
</tr>
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<td>35</td>
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<tr>
<td>40</td>
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<tr>
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<td>0.8819</td>
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<td>1.9790</td>
<td>1.8632</td>
<td>1.6107</td>
<td>1.2334</td>
</tr>
</tbody>
</table>

Note: The log(R/S) and log(E(R/S)) values for the KOSPI returns over the period 1980-2005 are estimated according to equation (6) and equation (9) respectively. The corresponding V-statistic estimated according to equation (11) is presented in the final two columns.
AR(1) residuals compensate for much of this problem, the results from R/S analysis dealing with high frequency data are questionable.  

The first step in Peters’ adjusted R/S analysis consists of dividing the total sample into sub-periods of equal length of 10 observations. The sample data may be split into 706 contiguous sub-periods of 10 observations. In equation (6), each range is then normalized by the standard deviation and the average (R/S)\_10 value across all sub-periods can be calculated in table 3. Figure 3 shows the log-log plot of the empirical R/S (R/S) values and the expected R/S [E(R/S)] values against log(n) for the daily KOSPI returns, based on the results of the R/S analysis in table 3. The E(R/S) values were calculated following equation (9).

In figure 3 the R/S values are indistinguishable from the E(R/S) values, while the end of the R/S slope shows two break points, one at 500 days (log(n)=2.7) and another at 1,000 days (log(n)=3). After 1,000 days, the slope rises again and deviates from that of E(R/S).

To investigate the deviation between the values of R/S and E(R/S), figure 4 presents a plot of the V-statistic against log(n) for the daily KOSPI returns. The values of V-statistic were computed following equation (11). Looking at the V-statistic plots in figure 4, it appears that the slope for n &lt;= 50 converges to that of the expected R/S values, and is not readily distinguishable from a random walk. However, during the period between 50 and 500 days, the slope appears to be increasing faster than that of the expected Hurst; the slope also increases dramatically between 500 and 1,000 days. After 1,000 days, the V-statistic flattens out, implying that the series

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10) To account for the potential effect of short-term dependence on the R/S statistic, Lo (1991) has sought to empirically demonstrate the robustness against short memory of a modified R/S analysis, deriving its limiting distribution under both short memory and long memory, and illustrating its power against certain long memory alternatives.

11) When doing regression in R/S analysis, a minimum starting point for a sample size is n &gt;= 10. The reason for this is that values of n &lt; 10 for the sample provide unstable estimates (Peters, 1994).

12) The total sample data for the KOSPI returns is divided by 25 divisors which are greater than or equal to 10: 10, 14, 20, 25, 28, . . . , 3,500. The Gauss program code used by Peters (1994, pp. 279-289) was modified to produce the estimates of empirical and expected Hurst exponent.
Figure 3  The log-log Plot of the Empirical $R/S (R/S)$ and the Expected $R/S [E(R/S)]$ for the Daily KOSPI Returns

Figure 4  The $V$-statistic for the Empirical $R/S (R/S)$ and the Expected $R/S [E(R/S)]$ against $\log(n)$ for the Daily KOSPI Returns
shows persistence up to 1,000 days, and then the memory dissipates. Furthermore, two breaks appear at \( n = 500 \) days (approximately two years) and at \( n = 1,000 \) days (approximately four years), indicating multiple non-periodic cycles in the daily KOSPI returns. \(^{13}\) \(^{14}\)

To assess the significance of these visually identified cycle lengths, the empirical Hurst exponent \( H \) and the expected Hurst exponent \( E(H) \) can be estimated over the entire period \( 0 \leq n \leq 3,500 \) and three sub-periods \( (50 \leq n \leq 1,000, \ 50 \leq n \leq 500, \text{ and } 700 \leq n \leq 1,000) \) by running the OLS regressions presented in equation (7). The significance test of equation (10) can then be applied to both the estimated \( H \) and \( E(H) \) values. Table 4 displays the results of the empirical \( H \) against the expected \( E(H) \) over the above periods. The interpretations of table 4 are, in order:

(i) The empirical \( H \) and \( E(H) \) coefficients can be estimated over the entire period \( 0 \leq n \leq 3,500 \). In this, the empirical \( H \) is 0.583, while \( E(H) \) is 0.556. The value for the significance test is 2.185, which being statistically significant supports a rejection of the null hypothesis of an \( i.i.d. \) series. This suggests that KOSPI returns exhibit the presence of long memory, indicating that the KOSPI market is an inefficient market.

(ii) In the first sub-period \( (50 \leq n \leq 1,000) \), the estimated \( H \) is 0.567 and the estimated \( E(H) \) is 0.539. The value of the significance test is 2.350, meaning that the empirical \( H \) is significantly different from \( E(H) \), suggesting a rejection of the null hypothesis of an \( i.i.d. \) series. This identifies a non-periodic cycle of four years in this sub-period.

(iii) In the second sub-period \( (50 \leq n \leq 500) \), the estimated \( H \) is 0.585 and the estimated \( E(H) \) is 0.547. The value for the significance test is 3.190, leading to a rejection of the null hypothesis. This sub-period exhibits evidence of long memory and a two-year non-periodic cycle.

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\(^{13}\) These cycles are estimated to be based on a 250-working-day year.

\(^{14}\) Technically speaking, multiple breaks (more than two) are indistinct in the \( \nu \)-statistics plot (Peters, 1994; McKenzie, 2001). Even if two small peaks are also observed at the point of 400 and 450 in figure 4, these points are excluded in this study.
(iv) In the third sub-period \((700 \leq n \leq 1,000)\), the estimated \(H\) is 0.933, indicating that the \(R/S\) values are increasing at a faster rate than a random walk process \((E(H)=0.517)\), which is statistically significantly different. This confirms the existence of long memory and the four-year non-periodic cycle in the daily KOSPI returns over this sub-period.

The findings suggest that the daily KOSPI returns appear to display evidence of long memory and possess two non-periodic cycles, one of a two-year duration and another of a four-year duration. The latter cycle corresponds to the Korean business or economic cycle.\(^{15}\) Figure 5 displays composite

\(^{15}\) After 1997 crisis, the Korean business cycle becomes shorter in figure 5. The nearest business cycle from 2005 is consistent with the former cycle, two year duration (Joyce, 2000; Lee, 2001).
### Table 4 Regression Results of R/S Analysis for Daily KOSPI Returns

<table>
<thead>
<tr>
<th>Periods</th>
<th>Whole</th>
<th>50 ≤ n ≤ 1,000</th>
<th>50 ≤ n ≤ 500</th>
<th>700 ≤ n ≤ 1,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>R/S</td>
<td>E(R/S)</td>
<td>R/S</td>
<td>E(R/S)</td>
<td>R/S</td>
</tr>
<tr>
<td>Constant</td>
<td>–0.107</td>
<td>–0.077</td>
<td>–0.026</td>
<td>–0.110</td>
</tr>
<tr>
<td></td>
<td>(0.012)**</td>
<td>(0.013)**</td>
<td>(0.0027)**</td>
<td>(0.011)**</td>
</tr>
<tr>
<td>Number of Points</td>
<td>25</td>
<td>25</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>Degrees of Freedom</td>
<td>23</td>
<td>23</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>Adj. R-square</td>
<td>0.998</td>
<td>0.998</td>
<td>0.999</td>
<td>0.999</td>
</tr>
<tr>
<td>Hurst Exponent</td>
<td>0.583</td>
<td>0.556</td>
<td>0.539</td>
<td>0.585</td>
</tr>
<tr>
<td></td>
<td>(0.050)**</td>
<td>(0.013)**</td>
<td>(0.003)**</td>
<td>(0.005)**</td>
</tr>
<tr>
<td>Significance</td>
<td>2.185**</td>
<td>2.350**</td>
<td>3.190**</td>
<td>34.96**</td>
</tr>
</tbody>
</table>

Note: Estimated results of equation (10) applied to the R/S and E(R/S) presented in table 3 over the sub-periods. Values in parentheses are standard errors. ** indicates the rejection of t-statistic at the 5% significance level.
indices of business indicators in the Korean economy. In this the vertical line represents each business cycle, and six business cycles appear between 1980 and 2005. The average Korean business cycle from 1980 to 2005 is equal to approximately four years (45.7 months). This average business cycle corresponds to the non-periodic cycle in the Korean stock market.

5. CONCLUSIONS

The long memory phenomenon in stock returns is an intriguing issue in financial literature, since the presence of long memory contradicts the assumptions of the EMH. This paper investigated the presence of the long memory property and non-periodic cycles in the Korean stock market.

The distributional properties of stock returns are important not only for the implementation of linear models, but also for determining the memory characteristics of the stock return process. It is apparent that KOSPI returns are very different from the returns that would meet the assumption of i.i.d. This suggests that KOSPI returns do not follow a random walk process, but instead follow a deterministic non-linear process associated with the presence of a long memory property. Linear statistical models based on the normality assumption, such as the CAPM and the Black-Scholes option pricing model, require some modification in order to more adequately reflect the distribution.

From the results of Peter’s adjusted R/S analysis, it is clear that the Korean stock market exhibits a high degree of persistence once short-term dependence effects have been removed by using AR(1) residuals. In addition, a non-periodic cycle of a duration of four-years is related to the business cycle in the Korean economy. The identification of a non-periodic cycle is not in agreement with the EMH. These long memory effects may be caused by heterogeneous investor horizons in the Korean stock market (Skjeltrop, 2000).

A broad acceptance of the presence of an observed long memory property
in stock returns remains incomplete in the long memory literature, because
the results are sensitive to the method of analysis and data chosen by
researchers (Crato, 1994; Kang, 2006; Howe, Martin and Wood, 1999). In
particular, Peter’s adjusted $R/S$ analysis is sensitive to short-term dependence,
which might bias the estimates of the Hurst exponent in the given sample
data (Ambrose, Ancel and Griffiths, 1993). To overcome this problem, there
is a need for further research that will investigate the presence or absence of
long memory using alternative methods and different sample data.

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